COMPTES RENDUS
(DOKLADY)
DE L'ACADÉMIE DES SCIENCES DE L'URSS

NOUVELLE SÉRIE

1944

IMPERIAL MYCOLOGICAL INSTITUTE
JUN 1947

VOLUME XLII

№ 9

ÉDITION DE L'ACADÉMIE DES SCIENCES DE L'URSS
MOSCOU

AVIS AUX AUTEURS

Les «Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS» paraissent tous les dix jours. Neuf numéros composent un volume pourvu d'une table des matières. Il y a quatre volumes par année.

Les «Comptes Rendus de l'Académie des Sciences de l'URSS» en de courtes communications précisent l'essentiel des travaux en préparation. Ces articles, ne dépassant pas en règle générale quatre pages, sont donnés en anglais, en français ou en allemand. Ils embrassent les sciences mathématiques, physiques, naturelles et appliquées, mettant au jour les résultats de recherches scientifiques en cours ou qui viennent d'être achevées, et représentant ainsi les plus récentes données de l'investigation scientifique.

L'auteur a droit à 50 exemplaires de son article.

Adressez les manuscrits à la Rédaction des «Comptes Rendus», Moscou, Wolkhonka, 14.

30 Mars

COMPTES RENDUS

(DOKLADY)

DE L'ACADÉMIE DES SCIENCES DE L'URSS

COMITÉ DE RÉDACTION

A. Arbusov, de l'Académie, A. Borissiak, de l'Académie, V. Chlopin, de l'Académie, A. Deborin, de l'Académie, A. Fersman, de l'Académie, A. Frumkin, de l'Académie, A. Kolmogoroff, de l'Académie (vice rédacteur), L. Orbeli, de l'Académie (rédacteur en chef), A. Richter, de l'Académie, S. Soboleff, de l'Académie, S. Wawilow, de l'Académie (vice rédacteur)

NOUVELLE SÉRIE

12-me année

Paraissant tous les dix jours

1944

VOLUME XLII, № 9

TABLE DES MATIÈRES

Pages

MATHÉMATIQUES

N. Erouguine. Sur les solutions fonctionnellement invariantes 371
S. Michlin. On the Convergence of Fredholm Series 373

ASTRONOMIE

A. Orloff, Corresponding Member of the Academy. On Variations of Greenwich Mean Latitude 377

PHYSIQUE

A. Kirpichev and I. Pomeranchuk. On the Theory of Transition Effects in Cosmic Rays 382
A. N. Murin. On Efficiency of Thermodiffusion Method for Separation of Isotopes 386
D. V. Skobeltzyn, Corresponding Member of the Academy. Density of the Stream of Particles in the Cores (Trunk Beams) of Auger Showers 388

CHIMIE APPLIQUÉE

I. I. Kitaigorodsky. Glass Cement Ceramics 393

AGROCHIMIE

Th. V. Chirikov and A. A. Maljughin. Phosphoric Anhydride as Assimilated by Plants from Sparingly Soluble Soil Phosphates 395

MICROBIOLOGIE

M. N. Meissel. Effect of Cancerogenic Substances upon Microbic Cell 399

SÉLECTION

A. N. Ipatjev. Accurate Morphological Methods for Studying Varieties 401

MÉCANIQUE DU DÉVELOPPEMENT

L. W. Poležajew. Phenomenon of Natural Apody in *Anura* and its Bearing on the Mechanics of Development 404

ENTOMOLOGIE

D. M. Fedotov. Some Observations on the Internal State of Imago of *Eurygaster integriceps* 408

Le présent numéro est suivi de l'indice au volume XLII.

Édition de l'Académie des Sciences de l'URSS
Moscou 1944

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLII, № 9

MATHÉMATIQUES

SUR LES SOLUTIONS FONCTIONNELLEMENT INVARIANTES

Par N. EROUGUINE

(Présenté par S. L. Soboleff, de l'Académie, le 7.IX.1943)

Considérons l'équation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=a^2\frac{\partial^2 u}{\partial t^2} \tag{1}$$

où a est une constante.

La fonction u est nommée solution fonctionnellement invariante pour l'équation (1), si $F(u)$ est une solution quelle que soit la fonction arbitraire F. Les solutions fonctionnellement invariantes de l'équation (1) s'expriment par la formule connue de Smirnoff-Soboleff

$$x+f(u)\,y+\frac{\sqrt{1+f^2(u)}}{a}\,t+\varphi(u)=0 \tag{2}$$

où $f(u)$ et $\varphi(u)$ sont des fonctions arbitraires (1).

S. Soboleff a démontré que l'équation (1) n'admet pas d'autres solutions fonctionnellement invariantes (2).

Considérons maintenant l'équation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=a^2\frac{\partial^2 u}{\partial t^2} \tag{3}$$

où a est encore une constante.

En tenant compte de la théorie générale des équations aux dérivées partielles, on peut démontrer que dans ce cas les solutions fonctionnellement invariantes s'obtiennent de la formule de Smirnoff-Soboleff

$$x+f(u)\,y+g(u)\,z+\frac{\sqrt{1+f^2(u)+g^2(u)}}{a}\,t+h(u)=0 \tag{4}$$

et encore dans la forme

$$u=\varphi\left(x+iz\sqrt{1+c^2}+\frac{cit}{a}\,;\,y+cz+t\,\frac{\sqrt{1+c^2}}{a}\right) \tag{5}$$

où c est une constante arbitraire et φ une fonction arbitraire.

L'équation (3) n'a pas d'autres solutions fonctionnellement invariantes. La formule de Smirnoff-Soboleff (4) contient toutes les solutions complexes fonctionnellement invariantes. La formule (5) contient seulement les soluttion complexes fonctionnellement invariantes, dont une partie coïncide avec certaines solutions complexes fonctionnellement invariantes, contenues dans la formule (4)

La partie commune des solutions complexes fonctionnellement invariantes, contenues dans la formule (4) et dans la formule (5), peut être exprimée ainsi:

$$x+i\sqrt{1+c^2}z+\frac{ict}{a}+\left(y+cz+\frac{t\sqrt{1+c^2}}{a}\right)f(u)+h(u)=0 \qquad (6)$$

où $f(u)$ et $h(u)$ sont des fonctions arbitraires. Ces solutions s'obtiennent de la formule (4) en posant

$$g(u)-i\sqrt{1+c^2}-cf(u)=0.$$

Remarquons encore que dans la formule (5) on peut changer les places de x, y et z.

Institut Mathématique Stekloff.
Académie des Sciences de l'URSS.

Manuscrit reçu
le 7.IX. 1943.

LITTÉRATURE CITÉE

[1] С. Л. Соболев, Труды Сейсмологич. ин-та, № 20 (1932). [2] С. Л. Соболев, Труды Физ.-мат. ин-та им. Стеклова, V, 259 (1934).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLII, № 9

MATHEMATICS

ON THE CONVERGENCE OF FREDHOLM SERIES

*By S. MICHLIN

(Communicated by S. L. Soboleff, Member of the Academy, 29.IX.1943)

It is known that the resolvent of Fredholm equation

$$\varphi(x)-\lambda\int_a^b K(x,\ s)\varphi(s)\,ds=f(x)$$

is a meromorphic function of λ. If the kernel $K(x, s)$ is bounded (more generally, if one of the iterated kernels is bounded), the resolvent may be represented as follows:

$$\Gamma(x,s;\lambda)=\frac{\Delta(x,s;\lambda)}{\Delta(\lambda)}$$

where $\Delta(x,s;\lambda)$ and $\Delta(\lambda)$ are integral functions of λ given by series of Fredholm

$$\left.\begin{aligned}\Delta(x,s;\lambda)&=\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}B_n(x,s)\lambda^n\\ \Delta(\lambda)&=\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}C_n\lambda^n\end{aligned}\right\}\qquad(1)$$

Here we have denoted

$$C_0=1,\quad C_n=\int_a^b\cdots\int_a^b K\begin{pmatrix}t_1,t_2,\ldots t_n\\ t_1,t_2,\ldots t_n\end{pmatrix}dt_1dt_2\ldots dt_n$$

$$B_0(x,s)=K(x,s),\quad B_n(x,s)=\int_a^b\cdots\int_a^b K\begin{pmatrix}x,t_1,\ldots,t_n\\ s,t_1,\ldots,t_n\end{pmatrix}dt_1\ldots dt_n,\quad n>1$$

$$K\begin{pmatrix}x_1,x_2,\ldots,x_n\\ s_1,s_2,\ldots,s_n\end{pmatrix}=\begin{vmatrix}K(x_1,s_1) & K(x_1,s_2)\ldots K(x_1,s_n)\\ K(x_2,s_1) & K(x_2,s_2)\ldots K(x_2,s_n)\\ \ldots\ldots\ldots\ldots\ldots\ldots\\ K(x_n,s_1) & K(x_n,s_2)\ldots K(x_n,s_n)\end{vmatrix}$$

The convergence of the series (1) is proved by means of the Hadamard evaluation of determinants.

T. Carleman (1) has demonstrated the convergence of the series (1) for all λ under the only condition that

$$\int_a^b \int_a^b |K^2(x,s)|\,dx\,ds = B^2 < \infty. \tag{2}$$

Here I will give a new demonstration of Carleman's theorem.

1. If $|\lambda| B < 1$, the resolvent is given by the power series

$$\Gamma(x,s;\lambda) = \sum_{n=1}^{\infty} \lambda^{n-1} K_n(x,s) \tag{3}$$

where $K_n(x,s)$ are the iterated kernels. We may suppose that $K(s,s)$ is summable in $a \leqslant s \leqslant b$. One can always obtain it by changing the values of the kernel on the diagonal $x = s$.

One can prove that $K_n(s,s)$ and $\Gamma(s,s;\lambda)$ are summable, and the equality

$$\int_a^b \Gamma(s,s;\lambda)\,ds = \sum_{n=1}^{\infty} \lambda^{n-1} \int_a^b K_n(s,s)\,ds \tag{4}$$

is true.

2. Let us consider the circle $|\lambda| \leqslant R$. We can put

$$K(x,s) = K'(x,s) + K''(x,s)$$

where $K'(x,s)$ is a degenerated kernel

$$K'(x,s) = \sum_{k=1}^{p} u_k(x)\,v_k(s)$$

and

$$\int_a^b \int_a^b |K''(x,s)|^2\,dx\,ds < \frac{1}{(R+1)^2}.$$

We shall denote the resolvent of the kernel $K''(x,s)$ by $\Gamma''(x,s;\lambda)$. In the circle $|\lambda| \leqslant R$ this resolvent may de represented by the series

$$\Gamma''(x,s;\lambda) = \sum_{n=1}^{\infty} \lambda^{n-1} K''_n(x,s).$$

Let us put

$$\tilde{u}_k(x,\lambda) = u_k(x) + \lambda \int_a^b \Gamma''(x,s;\lambda)\,u_k(s)\,ds$$

$$v_k^*(s,\lambda) = v_k(s) + \lambda \int_a^b \Gamma''(x,s;\lambda)\,v_k(x)\,dx.$$

$$a_{mk}(\lambda) = \int_a^b \tilde{u}_k(x,\lambda)\,v_k^*(x,\lambda)\,dx.$$

By $D_R(\lambda)$ we denote the determinant

$$D_R(\lambda) = \begin{vmatrix} 1-\lambda a_{11} & -\lambda a_{12} \ldots & -\lambda a_{1p} \\ -\lambda a_{21} & 1-\lambda a_{22} \ldots & -\lambda a_{2p} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ -\lambda a_{p1} & -\lambda a_{p2} \ldots 1-\lambda a_{pp} \end{vmatrix}$$

and by $\Delta_{km}(\lambda)$ the minor of the element located at the intersection between the k-th line and the m-th column. We can prove that the following relation is true:

$$\Gamma(x, s; \lambda) = \Gamma^{*}(x, s; \lambda) + \frac{1}{D_R(\lambda)} \sum_{k, m=1}^{p} \Delta_{km}(\lambda)\, \tilde{u}_m(x, \lambda)\, v_k^{*}(s, \lambda). \tag{5}$$

3. We put $x = s$ in (5) and integrate. Using the well-known integro-differential equation of the resolvent

$$\frac{\partial \Gamma(x, s; \lambda)}{\partial \lambda} = \int_a^b \Gamma(x, t; \lambda)\, \Gamma(t, s; \lambda)\, dt$$

we obtain

$$\int_a^b \Gamma(s, s; \lambda)\, ds = \int_a^b \Gamma^{*}(s, s; \lambda)\, ds - \frac{D_R'(\lambda)}{D_R(\lambda)}. \tag{6}$$

From (5) and (6) it follows that the poles of $\Gamma(x, s; \lambda)$ coincide in the circle $|\lambda| \leqslant R$ with the zeros of $D_R(\lambda)$.

Let λ' be a zero of $D_R(\lambda)$ of multiplicity q and $|\lambda'| \leqslant R$. From (6) it follows that λ' is a simple pole of the meromorphic function

$$\delta(\lambda) = \int_a^b \Gamma(s, s; \lambda)\, ds$$

with the residuum $-q$. Then

$$\Delta(\lambda) = e^{-\int \delta(\lambda)\, d\lambda} \tag{7}$$

is an integral function the zeros of which coincide in the circle $|\lambda| \leqslant R$ with the zeros of $D_R(\lambda)$ and have the same multiplicity. Using (5) we state that

$$\Delta(x, s; \lambda) = \Delta(\lambda)\, \Gamma(x, s; \lambda) \tag{8}$$

is also an integral function. It is sufficient to prove that the functions (7) and (8) are given by the series (1).

4. The quantities C_n and $B_n(x, s)$ satisfy the well-known relations

$$\left.\begin{aligned} C_{n+1} &= \int_a^b B_n(s, s)\, ds, \\ B_n(x, s) &= C_n K(x, s) - n \int_a^b K(x, t)\, B_{n-1}(t, s)\, dt \end{aligned}\right\} \tag{9}$$

Let us denote

$$A_n = \int_a^b K_n(s, s)\, ds.$$

Formula (9) easily yields the recurrent relation for C_n.

$$C_{n+1} = n! \sum_{k=0}^{n} \frac{(-1)^{n-k+1}}{k!} A_{n-k+1} C_k.$$

5. It follows from (4) that for $|\lambda| B < 1$

$$\delta(\lambda) = \sum_{n=1}^{\infty} A_n \lambda^{n-1}, \qquad \Delta(\lambda) = e^{-\sum_{n=1}^{\infty} \frac{A_n}{n} \lambda^n}$$

We can now obtain the power series for $\Delta(\lambda)$. This series will be convergent for all λ; since $\Delta(\lambda)$ is an integral function.

Let

$$\Delta(\lambda)=\sum_{n=2}^{\infty}\frac{(-1)^n}{n!}\gamma_n\lambda^n,\quad \gamma_n=(-1)^n\Delta^n(0).$$

We obtain, by (7),

$$\Delta'(\lambda)=-\delta(\lambda)\Delta(\lambda).$$

By Leibnitz formula

$$\Delta^{n+1}(0)=-\sum_{k=0}^{n}\frac{n!}{k!\,(n-k)!}\delta^{(n-k)}(0)\,\Delta^{(k)}(0)$$

or

$$\gamma_{n+1}=n!\sum_{k=0}^{n}\frac{(-1)^{n-k+1}}{k!}A_{n-k+1}\gamma_k.$$

The quantities γ_n and C_n satisfy the same recurrent relation, and $\gamma_0=C_0=1$. Then $\gamma_n=C_n$ for all n. Since the series (1) represents an integral function, this series must be convergent for all λ.

6. From the well-known integral equation for the resolvent

$$\Gamma(x,s;\lambda)=K(x,s)+\lambda\int_a^b K(x,t)\,\Gamma(t,s;\lambda)\,dt$$

it follows that $\Delta(x,s;\lambda)=\Delta(\lambda)\,\Gamma(x,s;\lambda)$ satisfies the equation

$$\Delta(x,s;\lambda)=\Delta(\lambda)\,K(x,s)-\lambda\int_a^b K(x,t)\,\Delta(t,s;\lambda)\,dt.$$

If we look for its solution in the form

$$\Delta(x,s;\lambda)=\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\beta_n(x,s)\lambda^n$$

we obtain

$$\beta_0(x,s)=K(x,s);\quad \beta_n(x,s)=C_nK(x,s)-n\int_a^b K(x,t)\beta_{n-1}(t,s)\,dt.$$

Formula (9) and the latter relations show that $\beta_n(x,s)=B_n(x,s)$ for all n. The integral function $\Delta(x,s;\lambda)$ is given by the series (1) and this series converges for all λ.

Khazakh State University.
Alma-Ata.

Received
28. IX. 1943

REFERENCES

[1] T. Carleman, Math. ZS., 9, H. 3/4 (1921).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLII, № 9

ASTRONOMY

ON VARIATIONS OF GREENWICH MEAN LATITUDE

By A. ORLOFF, Corresponding Member of the Academy

It is now fifty years since Chandler, a distinguished American astronomer, discovered two periods in the variations of latitude: one of 427 days, and another just one year long. After this discovery the interest in the problems of latitude variation increased considerably.

The International Service of Latitude was inaugurated, and a free cooperation of observatories joined in the work. The biggest observatories of the world: Pulkovo, Greenwich and Washington, installed excellent instruments which were all not alike, but rather built according to different designs, and with these instruments long term observations were carried out. In spite of this effort, however, no progress is to be recorded in the interpretation of the phenomenon after the brilliant start made by Newcomb, another American astronomer of high renown.

So that up to the present we have no theory on which it would be possible to forecast the successive positions of the pole and to arrange these data into an ephemeris.

On the other hand, the success of Chandler put many workers on the road of empiric research for new periods in the variation of latitude, and many such periods have indeed been reported but none of them rests on a theoretical basis or has found experimental support.

As one of the most important stages in the study of latitude oscillations in these forty years, one should point out the discovery of variations in the mean latitude of a given point of observation. Owing to it, astronomers are now compelled to return to the study of those slow, non-periodical or perhaps long-periodical changes of latitude which were registered by different students as long ago as in the seventies of the last century, and gave origin to doubts as to the constancy of latitude.

Chandler's disclosure of periodical oscillations directed the investigation of these questions along a different course. The study of slow changes of latitude have not been placed on a satisfactory foundation. On the contrary, it has been a general tendency with astronomers, while computing the coordinates of the pole, to stick at all cost to the same value of mean latitude for a given station throughout the period of observations, however long. There have been made attempts, it is true, to introduce the secular motion of the pole but no definite results have been secured in this way.

In 1925 I showed, in a paper [1] devoted to American astronomers, that if there are no secular terms in the variations of latitude, nevertheless the existence of slow wave-like changes with an amplitude up to 0″.1 is beyond doubt. These changes do not appear to be influenced by the motion of the pole and must therefore be eliminated from observations before one proceeds to the determination of its coordinates.

Table 1

$\Delta\varphi$, x and ψ_0 in $0''.01$

Date	$\Delta\varphi$	x	$\Delta\varphi - x$	ψ_0	Date	$\Delta\varphi$	x	$\Delta\varphi - x$	ψ_0
1911.7	+13	+13	0	—	1918.0	+ 8	+12	− 4	−0.4
.8	+15	+23	− 8	—	.1	− 2	+ 1	− 3	−0.6
.9	+15	+28	−13	—	.2	−10	− 8	− 2	−0.6
12.0	+13	+21	− 8	—	.3	−13	−14	+ 1	−0.8
.1	+ 6	+ 6	0	—	.4	−12	−16	+ 4	−0.8
.2	− 7	− 5	− 2	−5.2	.5	− 9	−10	+ 1	−0.8
.3	−19	−14	− 5	−5.1	.6	− 5	− 4	− 1	−0.6
.4	−22	−17	− 5	−4.4	.7	+ 1	+ 3	− 2	−0.2
.5	−20	−15	− 5	−3.8	.8	+ 4	+ 6	− 2	0.0
.6	−16	−12	− 4	−3.4	.9	+ 7	+ 8	− 1	−0.2
.7	−10	− 7	− 3	−3.2	19.0	+ 7	+ 9	− 2	−0.4
.8	− 5	− 1	− 4	−2.6	.1	+ 6	+ 6	0	0.0
.9	+ 1	+ 5	− 4	−2.1	.2	+ 4	+ 2	+ 2	+0.4
13.0	+ 6	+ 9	− 3	−1.8	.3	0	− 1	+ 1	+0.8
.1	+11	+10	+ 1	−1.7	.4	− 2	− 2	0	+0.9
.2	+13	+10	+ 3	−1.6	.5	+ 1	− 1	+ 2	+1.0
.3	+11	+10	+ 1	−1.4	.6	+ 3	− 1	+ 4	+1.2
.4	+ 7	+ 8	− 1	−1.2	.7	0	− 3	+ 3	+1.3
.5	+ 1	+ 4	− 3	−1.2	.8	− 7	− 7	0	+1.2
.6	−17	− 3	− 4	−1.3	.9	−10	− 8	− 2	+1.2
.7	−11	−10	− 1	−1.7	20.0	− 6	− 7	+ 1	+1.2
.8	−15	−14	− 1	−2.4	.1	− 2	− 3	+ 1	+1.2
.9	−17	−14	− 3	−3.0	.2	+ 2	0	+ 2	+1.2
14.0	−15	−11	− 4	−3.2	.3	+ 6	+ 5	+ 1	+1.6
.1	− 7	− 6	− 1	−3.0	.4	+10	+10	0	+1.9
.2	+ 1	+ 4	− 3	−2.9	.5	+14	+12	+ 2	+1.8
.3	+ 7	+14	− 7	−2.9	.6	+15	+12	+ 3	+1.8
.4	+12	+17	− 5	−2.8	.7	+13	+ 8	+ 5	+1.8
.5	+14	+16	− 2	−2.4	.8	+ 7	+ 1	+ 6	+1.8
.6	+10	+12	− 2	−2.2	.9	−10	− 8	− 2	+2.2
.7	+ 3	+ 4	− 1	−2.4	21.0	−16	−15	− 1	+2.6
.8	− 6	− 5	− 1	−2.5	.1	−14	−16	+ 2	+2.6
.9	−14	−14	0	−2.6	.2	−10	−12	+ 2	+2.2
15.0	−20	−20	0	−2.8	.3	− 2	− 4	+ 2	+1.4
.1	−23	−21	− 2	−2.5	.4	+11	+ 5	+ 6	+1.0
.2	−19	−14	− 5	−2.2	.5	+17	+13	+ 4	+1.0
.3	−10	− 3	− 7	−2.3	.6	+17	+16	+ 1	+0.6
.4	+ 1	+ 9	− 8	−2.8	.7	+12	+14	− 2	+0.4
.5	+16	+18	− 2	−3.5	.8	+ 5	+ 7	− 2	+0.6
.6	+28	+24	+ 4	−4.2	.9	− 5	− 4	− 1	+0.3
.7	+22	+23	− 1	−4.6	22.0	−15	−11	− 4	−0.4
.8	+ 9	+12	− 3	−4.5	.1	−17	−15	− 2	−0.5
.9	− 6	+ 1	− 7	−4.0	.2	−12	−14	+ 2	−0.4
16.0	−20	−12	− 8	−3.7	.3	− 6	−11	+ 5	−0.4
.1	−29	−20	− 9	−3.8	.4	− 4	− 2	− 2	−0.7
.2	−30	−24	− 6	−4.2	.5	+ 7	+ 8	− 1	−0.9
.3	−24	−21	− 3	−4.2	.6	+21	+18	+ 3	−1.0
.4	−12	− 9	− 3	−4.0	.7	+21	+22	− 1	−1.6
.5	+ 4	+ 4	0	−3.8	.8	+10	+14	− 4	−2.4
.6	+16	+17	− 1	−3.7	.9	0	+ 5	− 5	−3.2
.7	+23	+25	− 2	−3.6	23.0	−10	− 6	− 4	−3.8
.8	+22	+25	− 3	−3.6	.1	−18	−13	− 5	−4.4
.9	+13	+16	− 3	−3.4	.2	−23	−18	− 5	−5.0
17.0	− 6	+ 2	− 8	−3.0	.3	−24	−18	− 6	−5.6
.1	−19	−12	− 7	−2.8	.4	−20	−13	− 7	−6.2
.2	−26	−20	− 6	−2.8	.5	− 9	− 2	− 7	−6.8
.3	−26	−23	− 3	−2.6	.6	+ 8	+11	− 3	−7.2
.4	−17	−18	+ 1	−2.4	.7	+12	+19	− 7	−7.4
.5	− 3	− 6	+ 3	−2.0	.8	+ 8	+18	−10	−7.4
.6	+ 6	+ 6	0	−1.6	.9	0	+12	−12	−7.2
.7	+13	+14	− 1	−1.2	24.0	− 7	+ 2	− 9	−7.0
.8	+16	+17	− 1	−0.8	.1	−13	− 5	− 8	−7.3
.9	+16	+16	0	−0.4	.2	−16	−11	− 5	−7.6

Date	$\Delta\varphi$	x	$\Delta\varphi - x$	ψ_0	Date	$\Delta\varphi$	x	$\Delta\varphi - x$	ψ_0
1924.3	−17	−12	− 5	−7.6	1930.4	0	+ 2	− 2	−4.2
.4	−16	−12	− 4	−7.4	.5	+ 7	+10	− 3	−3.7
.5	−15	− 8	− 7	−7.0	.6	+10	+12	− 2	−3.6
.6	− 9	0	− 9	−6.6	.7	+ 9	+15	− 6	−3.9
.7	− 2	+ 6	− 8	−6.2	.8	+ 4	+ 8	− 4	−4.1
.8	+ 1	+ 9	− 8	−6.0	.9	− 4	− 2	− 2	−4.4
.9	− 2	+ 7	− 9	−5.7	31.0	−14	−10	− 4	−4.8
25.0	− 1	+ 5	− 6	−5.2	.1	−23	−16	− 7	−5.2
.1	− 2	+ 1	− 3	−4.6	.2	−21	−15	− 6	−5.6
.1	− 6	− 4	− 2	−4.3	.3	−16	−10	− 6	−5.6
.3	− 8	− 6	− 2	−4.2	.4	− 8	− 1	− 7	−5.8
.4	− 6	− 4	− 2	−4.0	.5	+ 2	+ 8	− 6	−6.1
.5	− 3	− 3	0	−3.8	.6	+ 9	+16	− 7	−6.0
.6	− 2	+ 1	− 3	−3.7	.7	+13	+20	− 7	−5.8
.7	− 2	+ 6	− 8	−3.5	.8	+ 6	+11	− 5	−5.6
.8	0	+ 6	− 6	−3.2	.9	− 4	+ 1	− 5	−5.6
.9	− 2	+ 4	− 6	−3.0	32.0	−13	− 7	− 6	−5.4
26.0	− 5	0	− 5	−2.7	.1	−18	−14	− 4	−5.0
.1	− 5	− 2	− 3	−2.3	.2	−20	−17	− 3	−4.4
.2	− 4	− 6	+ 2	−1.6	.3	−19	−13	− 6	−4.4
.3	− 6	− 6	0	−0.8	.4	−15	− 8	− 7	−4.7
.4	− 5	− 6	+ 1	−0.4	.5	− 3	0	− 3	−4.7
.5	− 1	− 3	+ 2	+0.2	.6	+11	+11	0	−4.6
.6	+ 3	0	+ 3	+0.6	.7	+12	+15	− 3	−5.0
.7	+ 4	+ 3	+ 1	+0.8	.8	+ 6	+15	− 9	−5.4
.8	+ 3	+ 4	− 1	+1.1	.9	+ 2	+ 9	− 7	−5.2
.9	+ 1	+ 2	− 1	+1.5	33.0	− 3	+ 1	− 4	−5.0
27.0	+ 3	+ 2	+ 1	+1.6	.1	−11	− 6	− 5	−5.2
.1	+ 3	+ 3	0	+1.4	.2	−20	−11	− 9	−5.3
.2	+ 3	0	+ 3	+1.4	.3	−19	−12	− 7	−5.0
.3	+ 4	0	+ 4	+1.4	.4	−14	−11	− 3	−4.4
.4	+ 5	0	+ 5	+1.0	.5	− 7	− 4	− 3	−4.2
.5	+ 2	+ 1	+ 1	+0.3	.6	+ 1	+ 4	− 3	−4.6
.6	+ 3	+ 3	0	0.0	.7	+ 8	+11	− 3	−4.6
.7	+ 5	+ 3	+ 2	−0.3	.8	+11	+13	− 2	−4.6
.8	0	+ 2	− 2	−0.9	.9	+ 8	+10	− 2	−4.5
.9	− 7	+ 1	− 8	−1.6	34.0	− 3	+ 4	− 7	−4.4
28.0	− 8	− 3	− 5	−2.0	.1	−12	− 4	− 8	−4.4
.1	− 6	− 5	− 1	−1.9	.2	−17	− 9	− 8	−4.5
.2	− 6	− 5	− 1	−2.1	.3	−17	−11	− 6	−4.6
.3	− 6	− 2	− 4	−2.2	.4	−12	− 9	− 3	−4.6
.4	− 2	0	− 2	−1.6	.5	− 4	− 2	− 2	−4.4
.5	− 4	+ 2	+ 2	−0.6	.6	0	+ 3	− 3	−4.1
.6	+ 5	+ 5	0	+0.2	.7	+ 2	+ 7	− 5	−4.1
.7	+ 1	+ 3	− 2	+0.8	.8	+ 5	+ 8	− 3	−4.6
.8	− 1	− 1	0	+1.2	.9	+ 4	+ 5	− 1	−5.4
.9	− 1	− 3	+ 2	+1.4	35.0	− 1	+ 2	− 3	−6.2
29.0	− 1	− 6	+ 5	+1.2	.1	− 7	− 1	− 6	−6.6
.1	− 1	− 6	+ 5	+0.9	.2	−11	− 1	−10	−6.2
.2	+ 2	− 2	+ 4	−0.6	.3	−12	+ 1	−13	−5.7
.3	+ 4	+ 3	+ 1	−0.3	.4	− 9	+ 3	−12	−5.6
.4	+ 2	+ 5	− 3	1.4	.5	− 4	+ 5	− 9	−5.8
.5	+ 7	+10	− 3	2.6	.6	0	+ 4	− 4	−5.6
.6	+11	+11	0	3.6	.7	+ 4	+ 2	+ 2	−5.2
.7	0	+ 9	− 9	4.5	.8	+ 1	0	+ 1	−4.7
.8	−10	0	−10	5.2	.9	− 6	− 3	− 3	—
.9	−17	− 8	− 9	5.3	36.0	−10	− 6	− 4	—
30.0	−21	−13	− 8	5.3	.1	−10	− 7	− 3	—
.1	−19	−15	− 4	5.4	.2	− 9	− 5	− 4	—
.2	−14	−10	− 4	5.4	.3	− 9	0	− 9	—
.3	− 8	− 4	− 4	4.9					

The coordinates of the pole so computed for a period of 48 years, from 1892 to 1940, have been published in an earlier paper of mine (2).

Such coordinates from their very definition involve merely short-periodical variations, and so the comparison of the observed latitudes with those computed from Kostinsky's formula

$$\Delta\varphi = x\cos\lambda + y\sin\lambda$$

must at once reveal the existence of slow changes of latitude whatever may be the factor that controls them.

Within recent years there have become known the variations of latitude deduced by Spencer Jones from observations made with a floating zenith-telescope at Greenwich for the period 1911—1936 (3).

If the changes in latitude $\Delta\varphi$ depended only upon the motion of the pole, we should have $\Delta\varphi = x$ at Greenwich, because there $\lambda = 0$. As it is, the differences $\Delta\varphi - x$ are also subject to the influence of local factors. In order to eliminate the influence of the z-term having an annual period and bring thereby better to show the slow variations of latitude, we have to derive the mean for the year from ten successive $\Delta\varphi - x$ differences given for every tenth part of the year. This mean we will denote by ψ_0. A certain constant quantity, the same for all values of ψ_0, can be discarded.

In two tables are given the values of ψ_0 for the Greenwich mean latitude, as determined by me from Spencer Jones' data. The columns of Table 1 contain from left to right: the date, the observed variation of latitude (2), the coordinate (x) of the pole (1), the difference $\Delta\varphi - x$ and the mean latitude ψ_0 calculated as described above, up to a certain constant.

Table 2

Mean u La titde, Observed ψ_0 and Calculated ψ_c in $0''.001$

Date	ψ_0	ψ_c	Date	ψ_0	ψ_c
1912.5	—38	—26	1925.0	—52	—53
13.0	—18	—27	.5	—38	—38
.5	—12	—27	26.0	—27	—19
14.0	—32	—25	.5	+ 2	— 1
.5	—24	—24	27.0	+16	+13
15.0	—28	—25	.5	+ 3	— 4
.5	—35	—28	28.0	—20	— 2
16.0	—37	—31	.5	— 6	— 6
.5	—38	—33	29.0	+12	—15
17.0	—30	—33	.5	—26	—26
.5	—20	—29	30.0	—53	—39
18.0	— 4	—21	.5	—37	—45
.5	— 8	— 9	31.0	—48	—50
19.0	— 4	+ 1	.5	—61	—52
.5	+10	+16	32.0	—54	—52
20.0	+12	+25	.5	—47	—50
.5	+18	+27	33.0	—50	—51
21.0	+26	+23	.5	—42	—50
.5	+10	+11	34.0	—44	—52
22.0	— 4	— 5	.5	—44	—55
.5	— 9	—25	35.0	—62	—58
23.0	—38	—44	.5	—58	—59
.5	—68	—58			
24.0	—70	—65			
.5	—70	—63			

The values of ψ_0 expressed in 0".001 are given again in Table 2 (second column) at half year intervals. By their side are the respective values of ψ_c computed from the empiric formula

$$\psi_c = a + 0.''030 \cos(2N + 240°) + 0.020 \cos(3N + 80°)$$

where N is longitude of the lunar node. For the first two series of Greenwich observations the value of a is put at—0".015; for the third (from 1927 to 1936), at—0".040.

This change in the time corresponds to the changes that occurred in the Greenwich instrument at the beginning of the third series of observations [(2), page 15].

The agreement between the computed (ψ_c) and observed (ψ_0) values of the mean latitude may be said to be satisfactory. However, the formula of ψ_c, as any other empiric formula, has no sense or value unless it is confirmed theoretically or by observations made at other points or at different dates.

Received
5. XI. 1943.

REFERENCES

[1] A. Orloff, Harmonic Analysis of the Latitude Observations, Astronomical Observatory, Odessa, 1925. [2] A. Orloff, C. R. Acad. Sci. URSS, XXXVII, No. 9 (1942). [3] H. Spencer Jones, Observations Made with the Cookson Floating Zenith-Telescope 1911—1936, Table 13.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLII, № 4

PHYSICS

ON THE THEORY OF TRANSITION EFFECTS IN COSMIC RAYS

By A. KIRPICHEV and I. POMERANCHUK

(Communicated by S. I. Wawilow, Member of the Academy, 8.VI.1943)

As well known, the transition of cosmic rays from a medium with a certain atomic number, z_1, to another medium with a diifferent atomic number, z_2, is accompanied with transition effects; these effects are dependent on the relation between the critical energy ε of the cascade theory of showers and the atomic number. The most important transition effect is the dependence of the number of showers on the thickness of the screen (Rossi's curve). The effect may be observable most frequently in the case of transition of cosmic rays from air to lead, since the transition effect is the higher, the greater the difference between the respective atomic numbers of the substances. Rossi's curves illustrate one of the most important quantitative regularities to which cosmic rays obey.

Directly upon its formulation ([1], [2]) the cascade theory of showers was able to account for a number of qualitative characteristics of transition curves. However, no accurate quantitative calculation of the transition curves, derived from the cascade theory of showers is as yet available. Rossi ([3]) carried out calculations of the transition curve on different assumptions with regard to the spectrum of the soft component in the air.

An analysis of this spectrum, as made in a previous work by the present authors ([4]), has shown its shape to be different from what has been assumed in ([3]); so the transition curves obtained in this work cannot be well founded. Landau ([5]) computed the relation between the maximum of Rossi's curve and its tail for the case of a transition curve due to the ionizing and the non-ionizing component. The relation found by this writer cannot, however, be directly compared to experimental data because: 1) no allowance is made for the fact that in the maximum showers are produced not only by air electrons and positrons, but by air photons, too.

It should be pointed out that this circumstance is essential when determining the total energy spent on the ionization of the soft component of the air at its transition into lead.

2) Use is being made of an erroneous distribution of δ-electronic showers in the air according to their size.

3) An incorrect correlation between the δ-electrons and electrons of decay in the air is made use of.

In the present work the shape of the transition curve beyond the maximum is being considered; at the same time evaluation is made of the relation between the maximum of Rossi's curve and its tail. The thickness of the screen has been chosen greater than those that served for the observations of the maximum.

Such a choice was due to the fact that at its maximum Rossi's curve is largely due to that part of the soft component of the air (or of any similar medium) whose energy E is of the order of the critical energy of the air. The spectrum of

the soft component within this region being but little investigated, an accurate calculation of the transition curve within the region adjoining the maximum involves considerable difficulties.

In the subsequent we shall analyse in detail the shape of the transition curve beyond the maximum. As for the relation of the maximum of the Rossi curve to its tail, it cannot be accurately determined for the same reason that prevents an accurate determination of the Rossi curve near the maximum (for ignorance of the spectrum of the soft component in air with $E \approx \varepsilon$).

Let us determine the dependence of the number of particles on the thickness of the screen recorded. When recorded experimentally [for example, under most perfect conditions by Rossi ([3]) and Spivak ([6])], the number of showers made up of more than two, or two and even one particle, with great screen thicknesses must show about the same dependence on thickness as the number of particles, the number of readings of the recording system being, as will be shown elsewhere, approximately proportional to the number of particles in a shower.

It should, however, be noted at once that the quantity measured by a system of counters is, strictly speaking, not proportional to the number of particles in a shower. A scrupulous analysis of what is to be measured should therefore precede the comparison between theory and experiment.

It may only be asserted that such experiments as are made with a recording system counting showers made up of a number of particles equal to or more than unity ([3]) are more appropriate for recording the number of particles than such experiments as imply the presence in a shower of two particles at least.

The number of particles $N(t)$ observed under the screen of a thickness t is determined with the aid of a differential spectrum of the soft component $P(E_0)dE_0$:

$$N(t) = \int a e^{\varphi(E,\,t)} P(E)\, dE. \qquad (1)$$

As usual([7]), $ae^{\varphi(E,t)}$ stands for the number of particles observed at a depth t in a shower starting with a particle of energy E; a varies according to whether $N(t)$ is the number of charged or uncharged soft particles; P may be the spectrum of the charged or uncharged soft component. If P refers to photons (P_γ), we shall obtain a transition curve due to the non-ionizing component.

If P refers both to photons and electrons ($+$ and $-$), the transition curve obtained is a complete one. Let us substitute for P the expression ([4])

$$P = \left\{ \frac{\varepsilon W_0^\gamma}{E^2 (rE + W_0)^\gamma} + \frac{\gamma \varepsilon W_0^\gamma r}{E (rE + W_0)^{\gamma+1}} \right\} \text{ const.} \qquad (2)$$

For photons const $= 0.52\, F_0$, for electrons const $= 0.4\, F_0$. The denotation is the same as in ([3],[4]); ε is the critical energy of air. F_0 is the full charged soft component in air.

Let us introduce new denotations: $E = \varepsilon e^y$; $W_0 = r\varepsilon e^{y_0}$

$$N = \text{const} \int a e^{\varphi} \left[\frac{e^{-y}}{(e^{y-y_0}+1)^\gamma} + \frac{\gamma e^{-y_0}}{(e^{y-y_0}+1)^{\gamma+1}} \right] dy. \qquad (3)$$

This integral is taken by «sattel-punkt's method».

Let us consider in more detail the integral from the first term in (3):

$$\int a e^{\varphi - t - \gamma \ln (1+e^{y-y_0})}\, dy = a(s_1) \sqrt{\frac{2\pi}{B_{y_1}}}\, e^{y_1 - y_0 - \gamma \ln (1+e^{y_1-y_0})} \qquad (4)$$

y_1 is determined from the equality:

$$\left(\frac{\partial \varphi}{\partial y}\right)_{y_1} - 1 - \frac{\gamma e^{y_1-y_0}}{1+e^{y_1-y_0}} = 0; \qquad \left(\frac{\partial \varphi}{\partial y}\right)_{y_1} = s_1$$

$$s_1 - 1 - \gamma \frac{e^{y_1-y_0}}{1+e^{y_1-y_0}} = 0. \qquad (5)$$

$$B = \frac{1}{t\lambda''(s_1)} + \frac{\gamma e^{y_1 - y_0}}{(e^{y_1 - y_0} + 1)^2}. \tag{6}$$

From (5) and (6) we obtain

$$y_1 - y_0 = \ln \frac{s_1 - 1}{\gamma + 1 - s_1}. \tag{7}$$

$$\frac{1}{\sqrt{B}} e^{\varphi_1 - y_1 - \gamma \ln (e^{y_1 - y_0} + 1)} = \frac{e^{\varphi(y_1, s_1) - y_0} (\gamma + 1 - s_1)^{\gamma + 1}}{(s_1 - 1)\gamma^{\gamma} \sqrt{\frac{1}{\lambda'' t} + \frac{(s_1 - 1)(\gamma + 1 - s_2)}{\gamma}}} \tag{8}$$

The second term in (3) can be considered in an analogous way

$$\frac{\gamma a(s_2) e^{\varphi(y_2, s_2)} (\gamma + 1 - s_2)^{\gamma + 1}}{(\gamma + 1)^{\gamma + 1} \sqrt{\frac{1}{\lambda''(s_2) t} + \frac{s_2 (\gamma + 1 - s_2)}{\gamma + 1}}} e^{-y_0}; \quad \lambda'' = \frac{d^2\lambda}{ds^2}. \tag{9}$$

Here

$$y_2 - y_0 = \ln \frac{s_2}{\gamma + 1 - s_2}. \tag{10}$$

The complete transition curve is obtained by summing up (8) and (9). The computation of $N(t)$ is to be carried out in the following way:

1) Let us choose arbitrarily values of s_1; 2) we obtain y_1 from formula (7); 3) with y_1 and s_1 known, t is found [for tables see (7)]; 4) from obtained value of y_1 and from given s_1 we find φ_1 (7). The assuming of various values for s_1 enables us to construct a curve which determines the first term of (3) at any thickness. A similar procedure applied to the second term in (3) gives us a second curve. Superimposing them, the transition curve $N(t)$ is ultimately obtained.

It should be noted that the quantity ae^{φ} of which we make use as of the number of particles in a shower, has a sense only for the number of particles in a shower, whose energy E is much higher than the critical one. The fact is that when determining ae^{φ}, ionization is not taken into account. It is quite evident, however, that the number of particles whose $E \gg \varepsilon$ ($E \geqslant 3-4\varepsilon$) as a function of t closely coincides with the number of showers as a function of t (a particle whose $E = 3-4\,\varepsilon$, transforms with a given t into a small shower after it has passed approximately one cascade unity of length). The two curves will be shifted with respect to each other by ~ 1 cascade unit of length*.

It should be noted that the theoretical curve is but little dependent on γ and r. From an analysis of the formulae (8) and (9) it follows that the principal rôle in the transition curve within a wide range of values of t belongs to the primary energies E_0 of the order $W_0 (y \sim y_0)$. As a lower limit for E we took $E = 2\varepsilon_{Pb}$; $\varepsilon_{Pb} = 8$MeV.

A good agreement between experiment and theory ([3],[6]) points to the fact that the experimental points refer to a magnitude proportional to the number of particles.

The probability of the records is proportional to the magnitude $\Pi (1 - e^{-\rho_k s_k})$ where ρ_k is the density of the particles at the point where the counter k is situated, s_k is its area. A more detailed analysis of this expression shows that with $t < 14 \div 15$, the magnitude measured in ([3]) and ([6]) is proportional to the number of particles in a shower. The respective calculations are published elsewhere.

Passing now to the determination of the relation between the maximum of the Rossi curve to its tail, we content ourselves here with an approximate

* This is sufficient to make it evident that the curve obtained in ([3]) and ([6]) should correspond to $N(t)$.

evaluation; more accurate calculations being reserved for a subsequent communication.

The number of showers q per one meson, due to δ-electrons in the tail of the Rossi curve in lead, is equal to $\frac{1}{24n}$ ([5]), n being the number of particles in one shower.

With $n=2$ [$n=2$ is playing the principal part in measurements of the type ([7])] we have $q=1/48$ in lead.

In the maximum of the Rossi curve we have showers due to air particles with an energy $\approx \varepsilon_A$.

In fact, the path of such showers with $E \approx \varepsilon_A \approx 2.5-3$ cascade units, which corresponds to the position of the maximum. The number of particles in such a shower equals $0.3 \frac{\varepsilon_A}{\varepsilon_{Pb}\sqrt{\ln \frac{\varepsilon_A}{\varepsilon_{Pb}}}} = \frac{0.3}{1.6} 12 \approx 2$.

The number of showers in the maximum is determined by the magnitude of the soft component in air, whose energy is of the order ε_A (including the photons). At the sea level the magnitude sought-for $\approx$ a half of the whole soft component per one meson is of the order of 30 per cent. Hence for the relation between the maximum and the tail we obtain $\frac{0.3}{1/48} = 14.4$. A more detailed analysis which is to be published elsewhere shows that this relation lies between 10 and 15. The experimental number ([6]) is equal to 7.

The discrepancy may possibly be caused by the presence of penetrating showers of non-cascade origin.

P. N. Lebedev Physical Institute.
Academy of Sciences of the USSR.
V. M. Molotov Erevan State University.

Received
8. VI. 1943.

REFERENCES

[1] H. Bhabha and W. Heitler, Proc. Roy. Soc. A, 159, 432 (1937). [2] J. Carlson and J. Oppenheimer, Phys Rev., 51, 220 (1937). [3] L. Janossi and B. Rossi, Proc Roy. Soc. A, 175, 88 (1940). [4] A. Kirpichev and I. Pomeranchuk, C. R. Acad. Sci. URSS, XLI, No. 1 (1943). [5] Л. Ландау, ЖЭТФ, 11, 32 (1941); Journ. of Phys., 4, 375 (1941). [6] П. Спивак, Journ. of Phys., 5.5 (1941). [7] L. Landau and G. Rumer, Proc. Roy. Soc. A, 166, 213 (1938).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLII, № 9

PHYSICS

ON EFFICIENCY OF THERMODIFFUSION METHOD FOR SEPARATION OF ISOTOPES

By A. N. MURIN

(Communicated by V. G. Chlopin, Member of the Academy, 2.X.1943)

The efficiency of the thermodiffusion process can be defined as the ratio of the free energy increment obtained on the separation of a gaseous mixture (and calculated for a standard temperature T_s) to the maximum work that can be materialized on the transfer of the heat expended in the separation from the heater (the heated wall of the separation tube) to the refrigerator (the cold wall).

Let us confine ourselves, for simplicity sake, to the case when one component of the gaseous mixture, say the heavy one, is contained in the mixture in small quantities, and the temperature difference between the hot and the cold wall is not great, compared to the absolute temperature of the gas, T. Suppose we have two gas filled reservoirs interconnected by a short tube whose length and cross-section area we denote by Δx and S, respectively. Between the reservoirs we set up a small difference of temperature, ΔT. Because of the temperature gradient there arises a flow of heavy molecules τ, equal to $S(D'n\Delta T / \Delta x - D\Delta n / \Delta x)$ where n is concentration of the heavy component, *i. e.* the number of heavy molecules in unit volume; Δn is difference of concentration in the reservoirs; D' and D are the coefficients of thermal and ordinary diffusion, respectively.

The flow τ creates an increase of free energy of the system

$$\frac{dF}{dt} = kT_s\tau\frac{\Delta n}{n} \qquad (1)$$

Here k is Boltzmann's constant.

The efficiency of the process is

$$\text{Eff} = \frac{dF / dt}{(dQ / dt)(\Delta T / T)} = \frac{kT_s S(D'n\Delta T / \Delta x - D\Delta n / \Delta x)\Delta n / n}{S\lambda(\Delta T / \Delta x)(\Delta T / T)} \qquad (2)$$

where dQ / dt is quantity of heat transferred from heater to refrigerator in unit time; λ is coefficient of thermal conductivity of the gas; T is absolute temperature of the gaseous mixture (strictly speaking, T is the temperature of the heater).

The efficiency varies as the system approaches the state of equilibrium, and is at maximum when

$$\frac{D}{D'\Delta T}\frac{\Delta n}{n} = {}^1/_2. \qquad (3)$$

In this case the relation (2) is reduced easily to the form

$$\mathrm{Eff}_{\max} = \frac{kn\,(D'/D)^2\,DT_s T}{4\lambda}. \qquad (4)$$

This maximum value of efficiency is useful as a criterion in estimating the fitness of a given chemical compound for the separation of isotopes by means of thermodiffusion. Besides, it may be used for comparing the separation process in the liquid and gaseous phase.

In the case of a gaseous mixture of isotopes

$$\frac{D'}{D} = \frac{\alpha}{T}; \quad \alpha \simeq 0.35\,\frac{m_1 - m_2}{m_1 + m_2}\,(^1); \quad \lambda \simeq 2.5\,\frac{C_v}{M}; \quad D \simeq \frac{\eta}{MC}. \qquad (5)$$

Here m_1 and m_2 are masses of the heavy and light molecule, respectively; η is viscosity; C_v is the grammolecular specific heat of the gas at a constant volume; M is molecular weight; C is number of grammolecules of the gas in unit volume.

By means of (5) the equality (4) is reduced to the form

$$\mathrm{Eff}_{\max} \simeq \frac{\alpha^2}{10}\,\frac{R}{C_v}\,\frac{T_s}{T}\left(\frac{c}{C}\right) \qquad (6)$$

where $\frac{c}{C}$ is relative concentration of the heavy component.

We know of only one certain case when isotopes were separated by means of thermodiffusion in the liquid — the concentration of heavy water carried out by Clusius and Dickel and repeated by Korsching and Wirtz. The results of experiments made by the latter authors can be applied to the evaluation of Soret coefficient D'/D in a water solution of D_2O. Making use of a formula derived earlier (²), we get

$$D'/D \simeq 0.7 \times 10^{-4}\,1/\text{degree}.$$

Orr and Butler (³) have measured the coefficient of diffusion of DOH in water (2—3 per cent solution) and found it to be equal to 4.75×10^{-5} cm²/sec. (at 45° C).

In evaluating the order of magnitude of the maximum efficiency one may put $T = T_s$. Then with the aid of the above-mentioned values for D'/D and D we obtain, according to (4)

$$\mathrm{Eff}_{\max} \simeq 0.4 \times 10^{-6}\left(\frac{c}{C}\right) \quad (T_s \simeq 320°)$$

In the case of the separation of hydrogen isotopes in the gaseous phase (H and D_2) the respective value for maximum efficiency is, according to (6)

$$\mathrm{Eff}_{\max} \simeq 0.4 \times 10^{-3}\left(\frac{c}{C}\right).$$

So we can state with full confidence that the concentration of the heavy isotope of hydrogen is, in point of energy, more profitably carried out in the gaseous phase than in the liquid.

If isotopes are separated in a tube by thermodiffusion, the maximum efficiency does not, of course, reach the value defined by the relation (4). Yet, it is easy to show that it can amount up to 0.7 of that value.

Radium Institute.
Academy of Sciences of the USSR.

Received
2. X. 1943.

REFERENCES

¹ W. Furry, R. C. Jones and L. Onsager, Phys. Rev., 55, 1083 (1939). ² A. Murin, C. R. Acad. Sci. URSS, XLI, No. 7 (1943). ³ W. Orr and J. Butler, Journ. Chem. Soc., 1273 (1935).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLII, № 7

PHYSICS

DENSITY OF THE STREAM OF PARTICLES IN THE CORES (TRUNK BEAMS) OF AUGER SHOWERS

By D. V. SKOBELTZYN, Corresponding Member of the Academy

In the preceding paper (1) I have suggested a method for the observation of «trunks» in Auger showers, based on the recording of coincidences in a group of several counters placed close one to the other. By means of it, as I am to show here, a very interesting problem receives a most simple solution; whereas the method usually employed and discussed by me in an earlier paper (2) is altogether unfit to solve it in a satisfactory way.

This problem consists in studying the space distribution of the stream of particles inside the trunk beam; or, in other words, in deciding the question as to the form of the relation $\rho = f(r)$ (where ρ is number of particles per unit area of cross-section) for very small distances r from the axis of the shower.

The data so far obtained on the distribution of particles in a shower along the radii r are based on the investigation of the relation $C = \varphi(D)$, where C is number of coincidences per unit time in two counters, and D is distance between the counters.

From the study of the curve $C = \varphi(D)$, however, no conclusions can be drawn as to the character of the relation $\rho = f(r)$, if r is below the range of 2 to 4 metres. This is owing to the fact that as the counter gets close to the axis of the shower; up to very small distances r; it is no longer sensitive to the increase of the density ρ; because several particles, when passing its effective cross-section simultaneously; are not told apart by the counter but are recorded as one.

Proceeding on the considerations, developed in my earlier paper (2); one may become convinced (if the law $\rho \sim 1/r$ holds) that: 1) the number of coincidences in the records of two counters ($\sigma = 200$ cm^2) increases but slightly (1. 5—2 times) as the distance D between them shortens within a range of 4 to 0 metres; 2) this increase is due to the softer components of the primary spectrums, so that the run of the curve $C = \varphi(D)$ brings out for small D not so much the relation $\rho = f(r)$ as the distribution of primary particles in the energy spectrum.

The truth of the latter statement would be greatly increased in the presence of a «pole» of a higher order; such as would correspond, for instance, to the relation

$$\rho \sim 1/r^2. \qquad (1)$$

My former computations (2) can be carried out without difficulty in this case as well. The result will show, in the first place, that the magnitude of the ratio $\frac{C(D=0)}{C(D=4\text{m})}$ depends essentially upon what is taken for the least limiting value of the radius $r_{\min}$ down to which (1) is valid, and, secondly, that the coincidences observed with $D = 4$ m and those observed with $D = 0$, are accounted for by utterly

different primary particles and by different regions of the spectrum of primary energies.

If the method of «multiple coincidences» ([1]) is made use of, definite conclusions on the character of the distribution of particles near the axis of the shower can be made by studying the dependence of the number of coincidences (C_n) registered in unit time upon the number of counters (n) involved in the scheme of coincidences and upon the axial section area (σ) of these counters.

As for the character of this dependence, two characteristic cases may be distinguished.

I. Density ρ of the current of particles registered by the counters is small, $\rho\sigma \ll 1$, and so is the probability p that an impulse will arise in a counter, $p \ll 1$. In this case p varies directly as σ.

Under these conditions the character of the distribution $\rho = f(r)$ has no influence on the form of the relation between the number of coincidences (C_n) in the records of n counters and the cross-section area σ

$$C_n \sim \sigma^n$$

So this dependence has its strength increasing with n. Moreover, the number of coincidences C_n sharply declines as n grows:

$$C_n \sim p^n \quad (p \ll 1).$$

II. Density ρ for a certain region of values $r < r_0'$ is very high, so that with r smaller than a certain r_0' the probability of registration is practically equal to one, and falls off rapidly for $r > r_0'$.

To schematize, we put $p = 1$ for $r < r_0'$, and $p = 0$ for $r > r_0'$. Then

$$C_n = \Phi \pi r_0'^2 \qquad (2)$$

where Φ is number of showers per unit time per unit area (we suppose that r is the same for every counter recording coincidences *).

As has been shown by my earlier computations, the action of Auger showers when the curve $\rho = f(r)$ descends abruptly and the number of counters n is large enough, fits very closely into the above-mentioned scheme, and the value r_0' is near to that value of r_0 for which one particle, on the average, crosses the section of a counter

$$\rho(r_0')\sigma = 1 \qquad (3)$$

(r_0' in (2) is equal to $k'r_0$ where k' is a coefficient varying with n but independent of σ).

So far as (2) is true, the relation to counters' cross-section area σ is completely determined (in contrast to the case I) by the dependence of r_0 upon σ. Since this dependence, according to (3), is in turn determined by the law of density decrease $\rho = f(r)$, we may come to some definite conclusions regarding this law through studying the influence of σ on the number of coincidences C_n. If, for instance, $\rho \sim 1/r$, then $C_n \sim \sigma^2$; for $\rho \sim 1/r^2$ we have $C_n \sim \sigma$, because $r_0 \sim \sigma$ in the first case and $r_0 \sim \sqrt{\sigma}$ in the second.

Similarly we find that by virtue of the shower having a trunk the dependence of C_n upon n is far from being pronounced (if n is large enough). Through studying this dependence it is open to us again to arrive at certain conclusions respecting the law $\rho = f(r)$.

---

* Formula (2) means that only those showers are registered—and these with a probability equal to one—whose axis is at a distance $r < r_0'$ from the counters.

Definite quantitative conclusions are possible if the computations, according to the scheme we used once before (1) for the case $\rho \sim 1/r$, be performed on the assumptions:

A. $\rho \sim 1/\sqrt{r}$

B. $\rho \sim 1/r^2$.

A.

$$\rho = \rho_0/r^{1/2}$$

where ρ_0 is determined from the condition

$$\int_0^R \frac{2\pi\rho_0 r\,dr}{r^{1/2}} = N.$$

N, total number of particles in the shower; R, its radius.

$$\rho_0 = \frac{3N}{4\pi R^{3/2}}; \quad r_0 = \frac{9N^2c^2}{16\pi^2R^2}$$

$$C_n = \int_0^R 2\pi r\,dr\,[1 - e^{-(r_0/r)^{1/2}}]^n.$$

Integration by parts carried out upon the substitution of variables leads to the separation of a series of terms containing factors of the form $e^{-k(r_0/R)^{1/2}}$; the development of these factors with respect to powers of $(r_0/R)^{1/2}$ readily gives the following result:

$$C_n = \pi r_0^2 \left\{ -\left(n - \frac{n(n-1)}{1\cdot 2}2^2 + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}3^2 - \ldots\right)\frac{R}{r_0} + \right.$$
$$+ \frac{2}{3}\left(n - \frac{n(n-1)}{1\cdot 2}2^3 + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}3^3 - \ldots\right)\left(\frac{R}{r_0}\right)^{1/2} -$$
$$- \frac{25}{72}\left(n - \frac{n(n-1)}{1\cdot 2}2^4 + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}3^4 - \ldots\right) +$$
$$+ \frac{2}{15}\left(n - \frac{n(n-1)}{1\cdot 2}2^5 + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}3^5 - \ldots\right)\left(\frac{r_0}{R}\right)^{1/2}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$+ \frac{(-1)^{k-1}}{1\cdot 2\cdot 3\ldots k}\left[1 + \frac{k}{3} + \frac{k(k-1)}{6} + \frac{k(k-1)(k-2)}{6}\right]\sum(k, n)\left(\frac{r_0}{R}\right)^{1/2(k-4)}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$+ \frac{1}{6}\left[-n\int_{(r_0/R)^{1/2}}^{\infty}\frac{e^{-x}dx}{x} + \frac{n(n-1)}{1\cdot 2}2^4\int_{2(r_0/R)^{1/2}}^{\infty}\frac{e^{-x}dx}{x} - \right.$$
$$\left.\left. - \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}3^4\int_{3(r_0/R)^{1/2}}^{\infty}\frac{e^{-x}dx}{x} + \ldots\right]\right\} \qquad (4)$$

where

$$\sum(k, n) = n - \frac{n(n-1)}{1\cdot 2}2^k + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}3^k - \ldots$$

The cross-section area of the counters enters into this expression through r_0.

The character of the relation between C_n and σ can be essentially different for different n. With small n it may correspond to relations typical of the case I; with large n we have the conditions corresponding to the assumption II. Which of the cases under consideration takes place for a given n, may be easily decided on the basis of the above formula.

The sums $\sum(k, n)$ enjoy the following property: $\sum(k, n) = 0$ for $n \geqslant k+1$.

It will be obvious that the coefficients before the terms involving the factor $(r_0/R)^{1/2(k-4)}$ are zeros up to the term with $k = n-1$. Of the remaining

terms the first, *i. e.* the one with $k=n$, will be of the highest power. If this main term (added with the factor πr_0^2) be examined with reference to the σ it involves, we shall find it always to correspond to the relation $\sim\sigma^n$ characteristic of the case I. On the other hand, the term containing the sum of exponential integrals is characterized (if $r_0/R \ll 1$) by the relation $\sim\sigma^4$ for any n, and is therefore to be classed with the case II.

One or the other of these limiting cases (either case I or case II) will take place if one of the two above-mentioned terms is essentially larger than the other.

Computations made on the assumption that $\sigma=200$ cm^2 ($R=70$ m) show that the conditions of the case I will hold up to $n=4-5$. But it is only when $n>7$ that the case II takes place.

B. $$\rho=\rho_0/r^3$$

$$\int_{r_{\min}}^{R}\frac{2\pi r\,dr\,\rho_0}{r^3}=N;\quad \rho_0=\frac{N}{2\pi\ln\frac{R}{r_{\min}}}=\frac{N}{14\pi}\ \text{for}\ \frac{R}{r_{\min}}=10^3\ *.$$

The condition $\rho r_0=1$ gives $r_0=\sqrt{\frac{N\sigma}{14\pi}}$

$$C_n=\int_0^R[1-e^{-(r_0/r)^2}]^n\,2\pi r\,dr=$$

$$=\pi r_0^2\left\{-\frac{1}{1\cdot 2}\left(n-\frac{n(n-1)}{1\cdot 2}2^2+\dots\right)\left(\frac{r_0}{R}\right)^2+\right.$$

$$+\frac{1}{1\cdot 2\cdot 3}\left(n-\frac{n(n-1)}{1\cdot 2}2^2+\dots\right)\left(\frac{r_0}{R}\right)^4$$

$$\dots\dots\dots\dots\dots\dots\dots\dots$$

$$\pm\frac{1}{1\cdot 2\dots k}\sum(n,\,k)\left(\frac{r_0}{R}\right)^{2(k-1)}$$

$$\dots\dots\dots\dots\dots\dots\dots\dots$$

$$+n\int_{(r_0/R)^2}^{\infty}\frac{e^{-x}}{x}\,dx-\frac{n(n-1)}{1\cdot 2}2\int_{2(r_0/R)^2}^{\infty}\frac{e^{-x}}{x}\,dx+$$

$$\left.+\frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}3\int_{3(r_0/R)^2}^{\infty}\frac{e^{-x}}{x}\,dx\dots\right\}\qquad(5)$$

With $r_0/R\ll 1$ the conditions corresponding to the case II are fulfilled for any $n\geqslant 2$.

Transforming (on the assumption that $r_0/R\ll 1$) the terms containing $\int\frac{e^{-x}}{x}dx$ in the manner indicated in my previous paper (1), we get for the sum C_n' of these terms the following expression (which holds for $n\gg m+1$):

$$C_n'=\pm\pi r_0^2\chi\left[2^m\frac{n(n-1)}{1\cdot 2}\ln 2-3^m\frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}\ln 3+\dots+(-1)^n n^m\ln n\right]$$

where $m=1$, 2 and 4 and $\chi=1$, 1 and 1/6 for $\rho\sim 1/r^2$, $1/r$ and $1/\sqrt{r}$, respectively.

By means of the above formulae the dependence of C_n on n and on σ can be taken into account for every of the considered assumptions with regard to $\rho=f(r)$.

* That is $r_{\min}=10$ cm for $R\simeq 10^4$ cm; $r_{\min}=10$ cm is chosen arbitrarily. Because of the logarithmic character of the relation, the arbitrary choice does not essentially affect the result

The results of computation are tabulated below. In the table the ratio C_3/C_6 is given, as obtained from formulae (4), (5) and (2)[1], and the dependence of C_n upon σ is indicated.

	$\rho \sim 1/r^3$	$\rho \sim 1/r$	$\rho \sim 1/\sqrt{r}$
C_3/C_6	1.5	4	55*
C_n^{**}	$\sim\sigma$	$\sim\sigma^2$ (for $n \geqslant 3$)	$\sim\sigma^n$ for $n \leqslant 4$; $\sim\sigma^4$ for $n > 7$

* After averaging with respect to the spectrum of primary ray energies, this ratio proves to be yet materially larger. The relations given in the first and second columns are independent of the primary spectrum.

** The figures of the last column have been calculated on the assumption that $(r_0/R) = 0.01$, which corresponds to $\sigma = 2 \times 10^2$ cm^2. For $(r_0/R) \ll 1$ the result is but insignificantly affected by the value of (r_0/R).

As is seen from the table, some definite conclusions on the run of the curve $\rho = f(r)$ in the region of small r follow immediately from the study of the relation of C_n to n and σ. If $\rho \sim 1/r^{1/2}$, then, according to the table, the characteristic peculiarities connected with the existence of a trunk in the shower do not become apparent with any degree of distinctness. In this case the probability of repeated coincidences due to Auger showers with n large enough to materialize the conditions corresponding to assumption II, would be so insignificant as to render their observation impossible. Yet, we can look upon this case as excluded.

For the future I have a plan for some observations to be carried out on the basis laid down in this paper and intended to secure data on radial distribution of particles in shower trunks *.

P. N. Lebedev Physical Institute.
Academy of Sciences of the USSR.

Received
6. XII. 1943.

REFERENCES

[1] D. Skobeltzyn, C. R. Acad. Sci. URSS, XLI, No. 2 (1943). [2] D. Skobeltzyn, ibid., XXXVII, 16 (1942).

* The conclusions of this and the preceding paper rest on the assumption that allowance for space fluctuations of density comes under the scope of Poisson's formula. The cascade mechanism does not appear to give any reason for doubts as to the applicability of this formula to the Auger showers observed after a cascade has crossed large thickness (in t units), even if the space distribution is considered without allowance for scattering. And as the distribution considered by us is a result of repeated scatterings, the law of Poisson may be applied to it with still greater certainty.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLII, № 9

CHEMICAL TECHNOLOGY

GLASS CEMENT CERAMICS

By I. I. KITAIGORODSKY

(Communicated by D. S. Beliankin, Member of the Academy, 24.XII.1943)

The quality of ceramic wares and their properties are determined by the chemical composition of the material they are made of; and to a very great extent, by their thermal treatment or burning. In the burning process three periods may be distinguished: heating or raising the temperature to a certain high optimum; keeping it up at this optimum level for a certain period of time; and cooling. Special importance is attached to the maximum of temperature and to how long it should be maintained.

According to the existing theory of burning; the proper choice of the temperature maximum has to provide for the formation, in the course of burning, of the requisite amount of liquid melt enveloping the particles of the mass. The duration of the period of maximum temperature should be such as to ensure a most complete recrystallization of the mass.

All of the present technology of the burning process rests entirely on this theory. The duration of the procedure in the factories is measured in days and weeks. Thus, in the case of moulded fire brick (chamotte) burning in a kiln of 100 m^3 capacity takes 6—7 d a y s and in the case of Dinas brick, 15 d a y s, 80 per cent of this time being expended on raising and lowering the temperature. This naturally implies an enormous expenditure of fuel.

Moreover, burning at high temperature being a long and complicated procedure, it is not always possible to produce wares with c o n s t a n t p r o p e r t i e s because of the temperature fluctuations in the gaseous medium. So a high ratio of spoiled and broken articles is taken in ceramic factories as a matter of course. On the principles adopted by the ceramic industry at this time it is extremely difficult to obtain a product of constant composition and in which the crystalline and vitreous phase would stand in the specific quantitative relation to each other. Apart from this, the empirically developed ceramic compositions include a high ratio of vitreous phase which is the weaker portion of a ceramic sample. The destruction of the material begins; as a rule; in the vitreous phase because of its comparative brittleness, lower thermal resistance and higher activity in corrosion

The theory of burning as we have it to-day is inadequate; looking back on many centuries of experience in manufacturic pottery, it presents merely an attempt to place a foundation under the empirically developed methods used in practice.

In my opinion; the two following principles should lie at the basis of a genuinely scientific theory of ceramic production.

1) It is necessary to secure a qualitative and quantitative constancy of the vitreous as well as crystalline phase in the product.

2) The proportion of the vitreous phase should be reduced to a minimum and assigned the rôle of a cement. The crystals of a ceramic specimen should be bound together by a microfilm of vitreous substance in which, just as in glass fibre, all the brittleness peculiar to the glass has vanished to give place to high mechanical qualities, and new factors have been created for the interaction between crystalline and vitreous phases.

These main principles of the glass cement theory of ceramics have been checked by me experimentally on different kinds of raw material. The experiments were carried out at Sverdlovsk (Urals Chemical Research Institute) and in Moscow (Mendeleyev Institute for Chemical Technology) in the years 1942 and 1943. At Sverdlovsk V. S. Udintseva, under my direction, has obtained and investigated samples of acid-proof ware made on the basis of felsites, aplites, Belkin and Lower-Uvel refractory clays, and marshalites.

The rocks reduced to powder, or, in the other cases, a mixture of clay and quartz sand, were blended with ground glass breakage (12 to 20 per cent). Previous to moulding the charge was moistened with water (5—8 per cent) and pressed. The moulded specimens were burned at 900 and 1000° C, and in the case of marshalite, at 1200° C. This rather low temperature was sufficient to melt the vitreous cement which softened at 600 to 620° C. It also secured the wetting and afterwards the adhesion of the solid phase.

To illustrate the results obtained some examples are referred to in the adjoined table and compared with the Chemical Industry Administration standard for acid-proof brick $\frac{\text{St}}{\text{ГХП}}\ \frac{15}{1515}$. The acid-proof specimens obtained by burning at 1000°C are higher in quality than the first grade acid-proof standard brick made from clays and chamotte and burned at 1250°—1280° C.

	Acid-proof standard	Acid-proof glass cement ceramics	
	$\frac{\text{St}}{\text{ГХП}}\ \frac{15}{1515}$	from aplite	from felsite
Composition of charge, %	Clay . . . 40 Refractory clay . . . 30 Chamotte . . 30	Aplite . . . . 80 Broken glass . 20	Felsite . . . . 80 Broken glass . 20
Burning temperature, °C	1250—1280	1000	1000
Absorption of water, %	8[1]—12[2]	1.95	3.36
Resistance to acids, %	95[1]—90[2]	99	99.45
Compression strength, kg/cm²	250[1]—150[2]	373	851
Thermostability, 15—500°C	no data available	No destruction after 20 changes of temperature	Withstood 19 changes of temperature

[1] First grade standard. [2] Second grade standard.

Laboratory specimens prepared on the same principle from clays and marshalite (at 1200° C), from slag (at 1000° C), from silica and corundum (at 1450° C) were all remarkable for high thermostability, had gained in mechanical strength and showed a low porosity.

The results of all work accomplished at Sverdlovsk, and now continued in Moscow, will be published later.

Received
24. XII. 1943.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLII, № 9

AGROCHEMISTRY

PHOSPHORIC ANHYDRIDE AS ASSIMILATED BY PLANTS FROM SPARINGLY SOLUBLE SOIL PHOSPHATES

By Th. V. CHIRIKOV and A. A. MALJUGHIN

(Communicated by D. N. Prianishnikov, Member of the Academy, 19.X.1943)

I. In the soil processes are taking place that lead to a dynamical equilibrium between its solid phase and the soil solution. Naturally enough, the equilibrium is not attained, being disturbed not only by physical-chemical processes, but also by micro-organisms and plants which consume the nutrient elements from the solution.

The soil solution is saturated with phosphate ions whose concentration is dependent on the form of the phosphates present in the soil. The root absorbs phosphate ions from the soil solution. The equilibrium thereby disturbed is restituted at the expense of the sparingly soluble soil phosphates which dissolve under the action of substances eliminated by the root, chiefly of carbon dioxide, and also owing to the changes in the content of the soil solution, due to absorption of nutrient elements by the roots ([1], [2]). The amount of soil phosphates dissolved by the root depends on the quantity of P_2O_5 absorbed by the plant within a certain period of time, in addition to the water-soluble P_2O_5 lost by the soil within the same period. This difference may serve to estimate the work expended by the root on dissolving the P_2O_5 from sparingly soluble forms.

Into the water extract there pass from soil mineral phosphates: phosphates of alkalies, acid phosphates of Ca and Mg, salts of sugar- and alcohol-phosphoric acids; and in small quantities also nucleins and nucleo-proteids. From organic phosphates the plant absorbs P_2O_5 only of those compounds of sugar- and alcohol-phosphoric acids that rapidly mineralize under conditions prevalent in the soil. The P_2O_5 of nucleins and nucleo-proteids which do not undergo hydrolysis under similar conditions remain in solution without being used up by the plant ([3]). The dissolving of mineral phosphates depends on the concentration of the hydrogen ions and calcium ions. It is therefore expedient to choose the relation between soil and water thus that the Ca-ion which is likely to pass from the soil into solution; should not precipitate the phosphate ions, or else that its effect in this direction be negligible. With a ratio soil : water equal to 1 : 25 the solubility of the P_2O_5 of the soil does not depend on the presence in the solution even of important quantities of the $Ca(NO_3)_2$ introduced. Therefore when applying water extracts in the case of the ratio mentioned above, comparable stable amounts are obtained.

The purpose of this study was to find out whether and to what extent P_2O_5 is assimilated by plants from sparingly soluble phosphates of the soil.

II. Under experiment were: 1) chernozem from the Voronezh regional experiment station; 2) podzolized loam soil from the forest of the Voronezh Agricultural Institute, and 3) sandy soil from the farm of the same Institute.

Overground Yield and P_2O_5 Content in Soils and Plants According to Periods

	Rich chernozem						Podzolized loam soil						Sandy soil		
	Buckwheat			Oat			Buckwheat			Oat			Buckwheat		
	prior to sowing	flowering	ripeness	prior to sowing	flowering	ripeness	prior to sowing	flowering	ripeness	prior to sowing	flowering	ripeness	prior to sowing	flowering	ripeness
1. Aboveground yield, mg	—	16.67	30.15	—	12.23	17.38	—	19.82	27.75	—	27.60	33.20	—	16.2	27.55
2. Content of water-soluble P_2O_5 per pot, in mg	33.7	31.0	14.6	33.7	28.8	11.9	72.3	48.1	24.1	72.3	43.3	18.7	313.6	212.8	177.6
3. Loss of water-soluble P_2O_5 within the period, in mg	—	2.7	16.4	—	4.9	16.9	—	24.2	24.0	—	29.0	24.6	—	100.8	35.2
4. P_2O_5 carried off by the plant per pot, in mg	—	43.0	65.0	—	28.5	59.0	—	60.0	88.0	—	70.5	82.0	—	153.0	200.0
5. P_2O_5 carried off by the plant within the period, in mg	—	43.0	22.0	—	28.5	30.5	—	60.0	28.0	—	70.5	11.5	—	153.0	47.0
6. Difference between the P_2O_5 carried off by the plant and the loss in water-soluble P_2O_5 (gr. 5—3), in mg	—	+40.3	+5.6	—	+23.6	+13.6	—	+35.8	+4.0	—	+41.5	−13.1	—	+52.2	+11.8
	Fallowing soil														
7. Content of water-soluble P_2O_5 per pot, in mg	33.7	31.5	19.1	33.7	31.5	19.1	72.3	47.6	48.3	72.3	47.6	48.3	313.6	294.4	256.0
8. Loss or increment of water-soluble P_2O_5 within the period, in mg	—	2.2	12.4	—	2.2	12.4	—	24.7	+ 0.7	—	24.7	+0.7	—	19.2	38.4
9. Total work of root within the period, in mg	—	+42.5	+18.0	—	+25.8	+26.0	—	+60.5	+ 4.7	—	+66.2	−12.4	—	+71.4	+50.2

All the pots were given KNO_3 at a ratio of 0.5 g N. The humidity was brought to 60 per cent to total capillary moisture capacity. The tests involved buckwheat and oat; the former was planted on May 14, the latter on May 8. The plants were gathered in part of pots at the stage of full flowering on July 11, in others at the stage of ripeness on August 14—15. After the plants were gathered, the soil contained in the pots was analysed for its content of water-soluble P_2O_5, Ca and NO_3 and its pH determined. At the same time the soil of the fallowing pots was also analysed. In this note data are considered that illustrate the content of P_2O_5 both in soils and plants. The results obtained with all kinds of soil are given in the table.

III. In the case of chernozem the content of water-soluble P_2O_5 shows a regular decrease throughout the growing period, both under plants and in fallowing soil. It should be noted that in the case of buckwheat the decrease is less pronounced than with oats, and still less pronounced is it in pots containing no plants. During the first period the decrease in pots containing no P_2O_5 both in soil under plants and in fallowing pots was found to be much less than during the second period.

By comparing the decrease in water-soluble P_2O_5 in the soil under plants with that in fallowing pots we become aware of the fact that in the former case the P_2O_5 disappears, as a rule, at a higher rate.

From a comparison of the amount of P_2O_5 carried away by the plant within a definite period with the water-soluble P_2O_5 lost by the soil within the same period, we learn 1) whether or not the loss in water-soluble P_2O_5 covers the requirement of the plant for P_2O_5 within the same period, and 2) whether the plant assimilated P_2O_5 from other forms of phosphates. The difference between the two figures referred to above may serve to evaluate the part played by the root in dissolving sparingly soluble soil phosphates. If this difference is a positive quantity, the plant should be taken to assimilate P_2O_5 of other forms aside from the water-soluble ones; if it is a negative quantity, this means that the water-soluble P_2O_5 has not been totally absorbed by the plant. Aside from the decrease in the P_2O_5 content of the soil due to its absorption by the plant, its content is liable to variations caused by microbiological and physical-chemical processes taking place in the soil.

The quantities thus obtained give us an idea as to the work of the root spent on dissolving the P_2O_5 of sparingly soluble phosphates of the soil in order to satisfy the plant's requirement for this substance, which will make up the lower level of the work. If the quantity obtained be added the amount of water-soluble P_2O_5 that has vanished from the fallowing soil, the sum will give us an idea of the upper level of the root's work. The actual sum must occupy an intermediate position since both processes are mutually interacting. As may be seen from the table, the values obtained for the upper level of the work did not change their sign, but the quantitative expression only.

IV. The second experiment was carried out with podzolized loam soil. The content of water-soluble P_2O_5 in loam soil under buckwheat and oats decreases regularly until harvesting. In fallowing loam soil a sharp drop is recorded at the first period, which is followed by no variations.

Within the period previous to flowering the roots of buckwheat have been found to have assimilated 35.8 mg of P_2O_5 taken from sparingly soluble phosphates; within the subsequent period the amount was as low as 4.0 mg. The roots of oats have assimilated during the first period from water-insoluble phosphates 41.5 mg of P_2O_5 while during the period that followed they did not take up the whole of the water-soluble P_2O_5 that disappeared from the soil.

The variations in the content in the soil of water-soluble P_2O_5 under oats and buckwheat assume a different course: the soil under the latter remains higher in water-soluble P_2O_5 than that under oats. This may be accounted for by the following: 1) the roots of oat are better absorber of P_2O_5 from solution than the roots of buckwheat; Domantovich (4), however, failed to find any difference in the way the two crops take up phosphates from solutions; 2) the roots of buck-

wheat affect more intensely the sparingly soluble soil phosphates than the roots of oats([5]); under buckwheat one can therefore observe a rapid restitution of the content of phosphate ions in solution; 3) the vigorous root system of the oat plant embraces the whole volume of the soil occupied by it and absorbs phosphate ions from solution much better than the buckwheat.

V. The experiment with sandy soil was carried out with buckwheat only, the results being the same as with chernozem and loam soil. The lower level of the work spent by the buckwheat's roots on dissolving sparingly soluble phosphates at the period previous to flowering was found to make up 52.2 mg. Within the period between flowering until ripening it is 11.8 mg of P_2O_5. The upper level makes up 71.4 and 50.2 mg of P_2O_5, respect vely. The content of water-soluble P_2O_5 in sandy soil was several times that in chernozem. Nevertheless, the largest amount of P_2O_5 assimilated by the buckwheat from water-insoluble phosphates was taken up by it when planted in sandy soil: 64 mg against 45.9 mg in chernozem, and 32.8 mg, in loam soil.

With all kinds of soil, by the end of the growing period, there is recorded a considerable residual amount of water-soluble P_2O_5. This amount attains with sandy soil 117.6 mg per pot, with loam soil, 24.1 mg and with chernozem, 14.6 mg. Thus, water-soluble P_2O_5 is the most fully used up when buckwheat is sown into chernozem.

Why then do the roots assimilate P_2O_5 from sparingly soluble compounds before they have taken up the whole of the water-soluble P_2O_5 in the soil? This phenomenon may be interpreted in the following way: 1) The roots affect but a small part of the soil particles that are in contact with them, and a small rizosphere, whereas the extract nvolves the soil as a whole; 2) According to observational data, a-water extract solves aside from mineral phosphates also the organic ones which are not assimitated by roots ([3]); and lastly 3) the loss of water-soluble P_2O_5 is restored at the expense of its passing from water-insoluble forms under the action of roots.

The data obtained can be summed up as follows:

1. Irrespective of the kind of soil they are growing in, both buckwheat and oat leave behind water-soluble P_2O_5. However, from one to two thirds of the total P_2O_5 taken up by plants from the soil is being absorbed by them from water-insoluble phosphates.

2. Buckwheat takes up the P_2O_5 of water-insoluble phosphates best of all when grown on sandy soil; next comes chernozem, and the last is loam soil.

3. Under oats grown in chernozem and loam soil the content of water-soluble P_2O_5 is lower than under buckwheat, which means that its roots absorb it to a fuller extent.

K. K. Gedroitz All-Union Institute
of Fertilizers, Agriculture and Soil Science.

Received
19. X. 1943.

REFERENCES

[1] Ф. В. Чириков, Журн. опытн. агрон., 15 (1914). [2] Ф. В. Чириков, там же, 17 (1916). [3] F. W. Parker, Soil Science, 24 (1927). [4] М. К. Доманто вич, Труды научн. ин-та по удобрениям, вып. 52, М. (1928). [5] Д. Н. Прянишников, Агрохимия, М. 1940. [6] G. V. Garola, L'engrais, II, Paris, 1925.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLII, № 9

MICROBIOLOGY

EFFECT OF CANCEROGENIC SUBSTANCES UPON MICROBIC CELL

By M. N. MEISSEL

(Communicated by L. A. Orbeli, Member of the Academy, 10.VI.1943)

Since the obtaining in a chemically pure state of substances liable to produce in number of animals transformation of normal cells into tumoral ones, it became possible to apply precise experiment to the study of the genesis of malignant growth and tumoral transformation. In this juncture the problem of the tumoral transformation of cells has acquired a more general biological import. The so-called cancerization of cells leading to the formation of tumours endowed with a malignant growth implies a co-existence of the following four conditions at least: a) more or less specific changes of a single cell or of a group of cells, stably transmitted to the next generations; b) ability of these altered cells to multiply progressively in the natural medium; c) the altered cells must possess some biological advantages as compared to the normal ones that would enable them to multiply more actively than the normal or the less affected cells; d) they must be able to overcome the barrier and regulatory mechanisms of the tissues and of the organism.

If the last three conditions are of more special significance, the first one may involve numerous cellular changes, not all of which satisfy the subsequent conditions, *i. e.* do not lead necessarily to the formation of a tumour.

Thus, the problem of cell alterations induced by cancerogenic agents is wider than that of tumour formation, and it rightfully requires a comparative study of the cell's reaction to cancerogenic treatment.

From this point of view particular interest should be attached to studies concerned with the effect of cancerogenic substances on the cells of tissue cultures, of *Protozoa* and micro-organisms ([1–7]).

I have tested the effect of benzpyrene, methylcholanthrene and ortho-aminoazotoluol upon cultures of the yeast organisms, *Saccharomycodes Ludwigii*, *Saccharomyces cerevisiae* and *coreanus*, *Nadsonia Richteri* and *Endomyces Magnusii*. Along with these, there were also taken saprophytic bacteria, both sporulating and non-sporulating ones. The cancerogenic substances were introduced in the nutrient medium in a water-colloidal state. The experiments involved both short-lived and prolonged cultivation (up to a year and a half and two years) upon media containing cancerogenic substances. In all cases without any exception there was active accumulation of these substances by the living cells of the micro-organisms. In Wood ultra-violet rays the developing cultures displayed an intense violet fluorescence, whereas the nutrient substrate with the solid particles of the cancerogenic substances suspended in it emitted a yellow-green fluorescence. According as the cultures grow older, the accumulation of the cancerogenic substances by the cells proceeds more intensely; it is especially intense in sporulating forms. The cancerogenic substances accumulated

by the cells are stably retained by them without undergoing any appreciable change in the process of fermentation. When studying this fluorescence, these substances are found to be transmitted in the process of multiplication to the next generations. Observations have shown that the bright-violet fluorescence of the cultures is due to the dissolving and accumulation of the cancerogenic substances in the protoplasm of the cells and in the lipoid inclusions. There is reason to believe that some of the cancerogenic substances, especially methylcholanthrene, are bound in the cells with the chondriosomes. When cultivating the fungus *Endomyces Magnusii* with methylcholanthrene, there is observed a peculiar gelatinization, thickenning and hypertrophy of the chondriosomes, owing to which the latter become clearly observable even *in vivo* in unstained cells. The hypertrophy takes place much at the expense of the proteinic part of the chondriome. The possible relation between the cancerogenic substances and the mitochondries has been pointed out by Graffi ([8]).

Being aware of the fact that the hypertrophy of the chondriosomes is usually connected with the so-called fermenting reconstruction of the cells ([9], [10]), we subjected these cultures to investigation in a Warburg apparatus. The energy of fermentation in methylcholanthrene cultures was found to be increased by 40 and oxygen consumption reduced by 26 per cent. Thus, even when cultivated for a short time on a methylcholanthrene medium, the cells acquire a morphological and physiological structure peculiar to the fermentative type. However, the peculiarities in the structure of the chondriomic apparatus, as well as the rather sharply depressed oxygen absorption make us believe that we are faced here with no common fermentative transformation, as that described by us for the transition from the aerobic conditions of life to those of fermentation, but rather with some pathological reaction that has not yet been closely investigated.

It is of interest to point out that the cell variations induced by benzpyrene and especially by ortho-aminoazotoluol are sharply distinguished from those described for methylcholanthrene. The more subtle mechanism underlying the cellular changes seems to be different for the various cancerogenic substances.

In 1933 the present author ([11]) found that a prolonged cultivation of microorganisms on media containing coal tar promoted the appearance of new heritable forms. Similar data have now been obtained with pure cancerogenic substances. These variations arise most frequently upon a prolonged cultivation (during 8—9 months). When such cultures are sown in separate Petri dishes, there appear colonies consisting of giant cells. Alongside of these, there appear others consisting of very small ones. The yeast organisms are found to vary sharply with respect to sporulation. Particular interest present forms of *Saccharomycodes Ludwigii* with a highly activized spore copulation taking place even in old cultures. All these alterations in cell and spore morphology are in many cases hereditary.

A detailed description of the cytology of the forms experimentally obtained will make the subject of a special paper.

Institute of Microbiology.
Academy of Sciences of the USSR.

Received
10. VI. 1943.

REFERENCES

[1] M. Des Ligneris, C. R. Soc. Biol., **120**, 777 (1935). [2] G. Mauer, Arch. exper. Zellforsch., **21**, 191 (1938). [3] Е. Лебензон и М. Магат, Мед. журн. УАН, 7, 381 (1937). [4] C. Dodge and B. Dodge, Ann. Missouri Botan. Garden, **24**, 583 (1937). [5] M. Wolman, Growth, **3**, 387 (1939). [6] J. Moltram, Nature, **145**, 184 (1940). [7] E. Cook, M. Hart and R. Jolly, Amer. Journ. Cancer, **35**, 543 (1939). [8] A. Graffi, ZS. Krebsforschung, **49**, 477 (1939). [9] M. Meissel, C. R. Acad. Sci. URSS. XX, 481 (1938). [10] М. Мейсель, Микробиология, 8, 381 (1939). [11] M. Meissel, Bull. Acad. Sci. URSS. sér. des sci. natur. et math., **6**, 849 (1933).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLII, № 9

PLANT BREEDING

ACCURATE MORPHOLOGICAL METHODS FOR STUDYING VARIETIES

By A. N. IPATJEV

(Communicated by A. A. Richter, Member of the Academy, 16.VII.1943)

The method of mean indices which is being widely used when describing varieties of cultivated plants is very convenient for testing and for ground control. It proves, however, insufficient either for studying varieties genetically or for breeding work with varieties.

The vast majority of the populations of cultivated plant varieties are not uniform as to their composition; they always display intravarietal variation which broach sometimes even the most rigid characters serving as a basis for intravarietal taxonomy.

Shakhaev (4) was the first to reveal for varieties of *Pisum sativum* L. the existence of some definite regularities underlying the qualitative composition of varieties (populations). After him this has been done by the present author for numerous varieties of truck- and fodder crops. It appeared that the «average individuals» within any variety or population (the so-called true-to-type, or pure-strain plants) which serve as a basis for describing a given variety and are kept as stock-material n seed selection, are invariably attended by strains for which we have co.ned the name of «constant components» (1, 2); these strains are as lawful a component of a given variety as the «average», or typical individuals.

Having recognized the fact of intravarietal polymorphism, breeders have long since elaborated a method of work within the species, which is known by the name of the method of «individual observations». This method consists in a thorough analysis of the separate individuals within a given variety and in a study of their development; this is mostly a purely morphological one, taking no account of whether they belong or not to the average type of a given variety. In this way the value of a given individual as a possible parent is established; then, by employing some efficient method of selection, the necessary plants are isolated as strictly as possible from among the others and can be used at will, either as parents in crosses, or as the original form in selection, as mentors, etc.

In some cases, as for example by Michurin in his breeding work with orchard trees (3), the individual approach to each separate plant has been brought to a very high degree. The individual approach to orchard plants as practised in his work may be compared to that of animal breeders to their stock-material.

From his investigation of the composition of varieties of truck, crops and orchard plants the author was brought to conclude that the usual morphological methods of testing varieties (comparison, standards, scales, weighing, measurements, etc.) are scarcely valid, without due account being taken of the developmental stage of a given plant. The comparison of the mean indices, *e. g.* length or incision of the largest leaf, or the expression of the characters of some

organ medium by order, as usually applied in the method of individual observations, proves also inadequate. The mean indices obscure those of the separate characters of individual organs, and the procedure fails, for it does not reveal the potency of the organism at the various stages of its development.

In our search for precise morphological methods for studying intravarietal variation we fixed our choice upon two of them, *viz.* upon the method of comparison of organs corresponding to one another, to which we gave the name of «comparative ontogenetic method», and upon the method of retracing the order of the plant's development. The latter serves as a supplement to the former one and may conventionally be called the «diagram method».

Fig. 1. 20. VIII. 1942. «Shakar-palak». Symbols: *I*. leaf; ♂, male flower; ♀, female flower; *III*. width of ovary in cm; *D*. length of ovary in cm

I. The comparative ontogenetic method consists in comparing the corresponding organs at the stage of their full development. Thus, for example, in a tomato plant (*Lycopersicum esculentum* Mill.) the fifth real leaves are compared, which have attained their maximum development, ceased growing and are liable to decay.

These leaves display various characters (length, width, incision, different petioles, etc.). As a rule, genetically closely related plants differ but slightly by those of their organs that appear first, so that the characters of these cannot be used to discriminate between individual plants. It is therefore unnecessary to record the distinctive features of all the organs (of leaves in our case), those of them which are clearly distinguishable being sufficient for the purpose.

After the fifth leaves have been compared, we proceed to a comparison between, say, the seventh, ninth, and so on. In the same way corresponding clusters, fruits or other organs are to be compared.

The application of the present method often permits to discriminate according to the mean types or constant components between morphologically closely related varieties or groups of components within a single variety.

II. The method of diagrams consists in recording the order of appearance of organs on a plant in the course of its development. The differences in the expression of a given organ are recorded on the same graph, along with the date of their determination.

Fig. 2. 24. VIII. 1942. «Amurskaya». Same symbols as in Fig. 1

The application of the method described is illustrated by the two graphs adjoined, which have been plotted for two individual plants of *Cucumis melo* L. To make them more demonstrative, sharply distinguishable plants have been taken, belonging to different varieties: «Amurskaya» and «Shakar-palak».

The graphs (schemes) clearly bear out the difference in the arrangement of male and female flowers along the axes of branching, in the mode of development, number of leaves and flowers. The schemes can be made still more accurate by plotting the dimensions of separate parts of the plants, which we have done for fruits only. The dates should be given in order to enable to judge from the graph of the actual earliness of the plant.

S. M. Kirov Agricultural Institute at Omsk.

Received 16. VII. 1943.

REFERENCES

[1] А. Н. Ипатьев, Доклады ВАСХНИЛ, № 9 (1937). [2] А. Н. Ипатьев, Труды Омского с.-х. ин-та, XVII (1939). [3] И. В. Мичурин, Итоги 60-летних работ, 1936. [4] Р. Шахаев, Полевой определитель сортов главных с.-х. растений, культивируемых в совхозах Сахаротреста, Киев, 1930.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLII, № 9

DEVELOPMENT MECHANICS

PHENOMENON OF NATURAL APODY IN *ANURA* AND ITS BEARING ON THE MECHANICS OF DEVELOPMENT

By L. W. POLEŽAJEW

(Communicated by L. A. Orbeli, Member of the Academy, 18.XI.1943)

In the course of an examination of several thousand tadpoles of *Rana temporaria*, collected during summer 1938 in the vicinity of the Kropotov biological station (Kashira) for experiments on limbs, several animals were found to lack

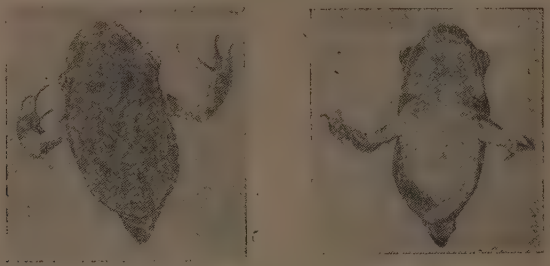

Fig. 1. Apodic tadpole of *Rana temporaria*, brought up to the end of metamorphosis. Fore limbs compensatory hypertrophied. a) Viewed from above, b) from below

completely one or, more frequently, both posterior limbs. A close examination of these animals led us to the conviction that we were confronting here cases of congenital apody. On the place where the limb is normally formed, no stump was present, the place was quite smooth, the epithelium was inflated and separated from the inner tissues.

Moreover, the monstrosities were detected at such a stage when the limbs could easily have regenerated after mutilation.

The inhibition of the development of the limbs could hardly have taken place at the stages of gastrulation and neurulation, when the regeneration capacity of the material forming the limb is very high, or at the stage of the lens-like anlage of the limb, which is capable of autonomous development even after its transplantation.

As in some of the cases the development of only one of the posterior limbs was inhibited, it is scarcely probable that this inhibition was due to a lack of material able to form the limb. Most probable is the supposition that the monstrosity was caused by a disturbance of the relation arising between the epithelium

and the mesenchyme cells proliferating from the thickening of the somatopleura in the region of the limb.

This is what actually occurs in those cases when the development of the limb is completely suppressed in the experiment ([1], [2], [5], [12]).

Microscopical investigation confirmed the probability, of the above supposition. The apodic animals were reared to different stages of metamorphosis up to its full completion (Fig. 1) and then fixed and studied histologically. At early stages the epithelium was found to become separated from the complex of inner parts which possess traits showing that they undoubtedly belong to the limb. This is especially clear when later stages are fixed. These parts consist of atypical skeletal rudiments, blood vessels, nerves and morphological blood

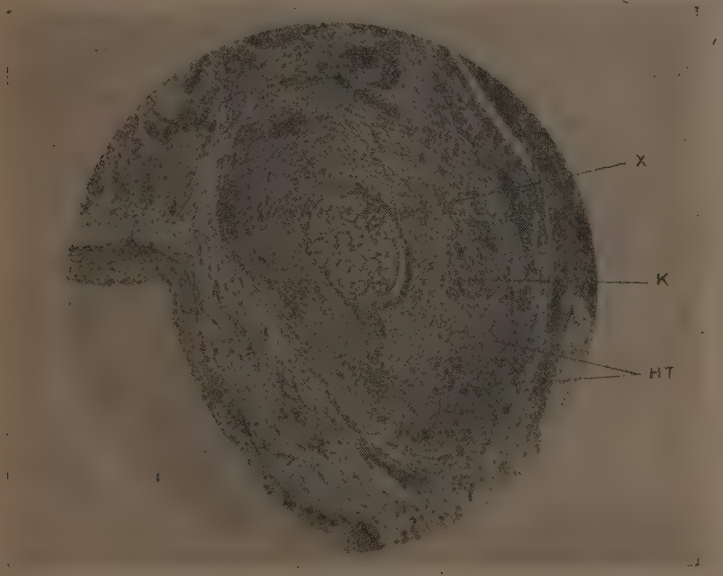

Fig. 2. Frontal section across the region of hind limbs in an apodic animal. *X*, cartilage *K*, bone; *FP*, formal blood elements; *HT*, amorphous tissue of mesenchymal origin

elements. Short pieces of the skeleton consist of osseous and cartilaginous tissues and are covered with a periost and a perichondrium. In their cavity are present elements of the marrow and morphological elements of the blood (Fig. 2).

All this complex of tissues is to be regarded as an atypically developed pelvis and parts of the femur.

We might limit ourselves to the above conclusion about the cause of apody in our cases. However, they are of interest in some other respects.

In the normal ontogenesis the following succession of events is observed: first of all, an anlage of the organ appears (organological differentiation), further its histological differentiation takes place, and lastly it undergoes cytological differentiation. For example, during the development of limb at first there arises a cytologically undifferentiated bud, then in the anlage skeletal and muscular tissues appear, and later the cells of bones, cartilage and connective tissue and striated muscular fibres are differentiated in them.

In our cases the tissues and cells are highly differentiated in the absence of the development of the limbs. In other words, the histological and cytological

differentiation proceed without an organological one.

The different forms of differentiation become separated from each other. Phenomena of this kind may undoubtedly be observed in other cases, as, *e. g.*, in the above-cited experiments on the inhibition of the development of limbs ([1], [2], [5], [12]).

However, the authors had here other aims before them and fixed the material very early, when the histological and cytological differentiation was not completed. The above-mentioned phenomenon escaped therefore their attention. Nevertheless, the fact of separation of different forms of differentiation established here finds confirmation in our data on the regeneration of limbs in amphibia, particularly in the phenomenon of transformation of prochondral aggregations into cartilaginous columns in the regeneration anlage, whose organological differentiation has been inhibited ([6]), and in other data ([3], [7]). In general, the possibility of a stage falling out without influencing the final link of morphogenesis has convincingly been demonstrated by the late D. P. Filatow in his excellent experiments on the formation of the lens from the ectoderm of the gastrula ([3]).

It would be erroneous to think that the organological, histological and cytological differentiation are independent on one another. First, as has already been noted above, in the normal ontogenesis the differentiations arise in a definite sequence. Secondly, experiments show that the organological differentiation depends on the histological one: a regeneration anlage of a limb the development of which is inhibited, fails to develop or develops very atypically, if its histological differentiation is considerably advanced ([4]). Thirdly, the organological differentiation depends on the cytological one: the formation of the anlage is always conditioned by definite properties of the cells of which it consists.

Therefore we may conclude that the following chain of events exists in reality: the initial stages of organogenesis (organological differentiation) are based on the existence of a certain cytological differentiation and determine the histological differentiation, and the latter determines a new cytological differentiation.

The occurrence of histological and cytological differentiations without an organological one is explained by the existence of conditions similar to those present in case the organological differentiation determines the histological one.

The same may be said about the interrelations of the histological and cytological differentiations. The significance of the separation of different forms of differentiation and of the possibility of the existence of one of them when the preceding one is excluded, lies particularly in the possibility of an experimental approach to the origin of the given differentiation.

Apodic tadpoles of *Rana temporaria* were found by us not only in the summer of 1938, but in 1939 and 1940 as well, and always in one and the same reservoir. It is hardly probable that this reservoir may offer some peculiar conditions causing apody. It is more probable to suppose that from year to year a definite pair of frogs or at least one certain frog spawned there and that apody is thus a hereditary trait, manifested in the case of a definite combination of genes. If this is so, we have here a case of morphogenesis, where not only its mechanics of development can be known, but its hereditary basis as well; and where a direct relation can be found between morphogenetical factors and heredity. Such cases are at present exceedingly rare, so that only a few of them are known ([8]—[11]) and present undoubtful interest not only for the mechanics of development and genetics, but for the theory of evolution as well. Cases of hereditary apody are lethal under certain ecological and other conditions and are eliminated by natural selection, but under some conditions they may have a positive adaptive value and may become evolutionally fixed and give rise to a sudden appearance of new forms.

In this connexion it is interesting and important to note that this result may be caused not by a loss by the cells and tissues of their general morphogenetic capacity, but by a disturbance of some simple correlation of parts in the ontogenesis.

In our cases, for example, the connexion between the epithelium and the mesenchyme cells is disturbed, leading to the inhibition of the formation of the limb anlage. An evolutionary restoration of such a lost simple connexion may be relatively easy and may lead under certain conditions to the reappearance of forms very close to the ancestral ones.

It is possible that the appearance of a number of apodic forms of animals from ancestors possessing limbs (snakes, apodic amphibia, whales, walruses, seals and other animals) is based on such a hereditary variation which, without inhibiting completely the morphogenetical capacity of the cells, results in a disturbance of some simple correlations determining the initial stages of organogenesis. This assumption can be checked by direct experiments.

Institute of Cytology, Histology and Embryology.
Academy of Sciences of the USSR.

Received
19. XI. 1943.

REFERENCES

[1] D. Filatow, Roux' Arch., **110** (1927). — [2] D. Filatow, ibid., **121** (1930). [3] Д. П. Филатов, Биолог. журн., **6** (1937). [4] I. I. Morozov, C. R. Acad. Sci. URSS, XX, No. 2—3 (1938). [5] L. W. Poleżajew, ibid., IV (XIII), No. 8 (1936). [6] L. W. Poleżajew unter Beteiligung von W. N. Faworina, Roux' Arch., **133** (1935). [7] L. W. Poleżajew und G. I. Ginzburg, C. R. Acad. Sci. URSS, XXX, No. 6 (1941). [8] I. I. Schmalhausen, ibid., II, No. 5 (1934). [9] И. И. Шмальгаузен, Труды ин-та зоол. та біол. ВУАН, I (1934). [10] И. И. Шмальгаузен, ibid., VIII (1935). [11] И. И. Шмальгаузен, ibid., XVII (1937). [12] J. Schwind, Journ. exp. Zool., **63** (1932).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1944. Volume XLII, № 9

ENTOMOLOGY

SOME OBSERVATIONS ON THE INTERNAL STATE OF IMAGO OF *EURYGASTER INTEGRICEPS*

By D. M. FEDOTOV

(Communicated by A. A. Borissiak, Member of the Academy, 14.II.1943)

The present note contains a brief account of the observations concerned with the behaviour of the internal organ systems of the imago *Eurygaster integriceps* Put. throughout the period of the bug's imaginal stage, from its metamorphosis into an imago and until natural death. The considerable extent of the imaginal stage in this insect, which embraces all seasons, makes the study of the behaviour of its internal organs at various seasons rather promising both in view of a better insight into the high adaptation of this insect, and for the purpose of its control. This article sets forth preliminary data of a work that has been carried out by me in the Kashka-Darya region of Uzbekistan in the course of the expedition sent by the Academy of Sciences of the USSR (October 1941—September 1942). The work represents the first step in the study of the physiology of *Eurygaster integriceps*.

The mediterranean character of the climate prevailing in the Kashka-Darya region, the presence there of vast valleys and mountain massifs, the abundance and diversity of the nature, especially of vegetation, all this creates conditions favourable for *Eurygaster integriceps*. According to the data of the meteorological station at Kitab, the annual temperature averages +14.2°C; that of the coldest month, which is January,—2.2°; of the warmest, July, +28.9°. There is scarcely any precipitation within June—September. The summer is hot, arid, with low air moisture (20—30 per cent). Winter is mild, inconstant, showing occasionally sudden, though insignificant drops of temperature. The conditions are thus favourable for the imago bugs even during the passive period of their life. The imaginal stage of the insect lasts in the Kashka-Darya region from the first fortnight of June till the end of April—beginning of June of the next year, according to the time of spring awaking. The time when the bugs leave the crops and fly over to the mountains for aestivation, which occurs during the second fortnight of June, is dependent on their internal condition. They take no less than a week to get ready for the flight. During this period they have to attain a definite degree of hardness of their external chitin and of the chitinous linings of some of their organs; to prepare their odoriferous glands, complete the development and fortify their thoracic muscles, wings and legs and their joints with the external skeleton. They have also to accumulate a copious fat body filling their abdomen, and the first division of the midgut is to be filled with food. The excretory system is intensely working out excreta that are eliminated by the rectum together with fecalia. The sex glands remain underdeveloped.

After migrating from the valley crops to the foothills and mountains, the bulk of the insects of the new generation compensate for the loss suffered by their organisms during the flight. They are feeding either on barley and wheat crops of the foothills and mountains, or on wild plants. To the reserve of food accumulated when feeding on the valley crops, there is added either a mass of starch from barley and wheat, or coloured sap and oil-drops taken from wild plants. The consumption of the fat body seems to be insignificant, but the odoriferous glands are being consumed intensely: the deep-orange they displayed in bugs when feeding on valley crops fades now into a pale-orange. According to observation, the bugs look for a place for aestivation and gather together, being allured by the odour of their odoriferous glands. During the flight there is also spent a part of haemolymph. The bugs become therefore less fleshy, and their salivary glands loose the turgor which is typical of them before they leave the valley crops. According to the state of the internal organs, in the first place of the odoriferous glands, it is no difficult matter to distinguish on the crops of the foothills the bugs that have newly migrated there from the local ones, *i. e.* such as are being developed in place. This fact may be of practical value.

The period of aestivation which starts at the second half of June is characterized by an insufficient air humidity (June—September). The amount of the haemolymph decreases, the salivary glands become flabby, the intestine smaller in volume. It does not cease functioning, however, though slowly. The midgut slowly assimilates the food stored. The 1st division of the midgut changes but little, while the 2nd and the 3rd ones become thinner, and the 4th one transparent. In the rectum a small quantity of thick black fecalia is accumulated. The coil of Malpighian tubes which is yellowish in bugs previous to their leaving the crops, grows dark-green; in connexion with the sluggish activity of the alimentary canal, the state of the excretory system varies but little: the pylorus and the main trunks of Malpighian tubes remain brownish in colour, or become transparent because of the elimination of the excretion. The odoriferous glands are renewed, and become intense orange again. The ovary remains underdeveloped, in the form of white short tubules; the testes, though retaining their spatular shape, show, however, appreciable pigmentation. The aestivation of the bugs cannot thus be considered as a period of perfect passivity. Their state during aestivation facilitates their transition to autumnal activity (which occurs within middle September to middle October in Kirghizia), when they seem to leave places that are not fit for a lengthy hibernation and look for more convenient ones. The odoriferous glands become an intense orange again, their activity being intense now; the bugs exhale a strong odour, by which the search for a place convenient for aestivation is promoted. By way of an increased ramification of the abdominal tracheae, evacuation of the rectum from fecalia, and of the Malpighian tubes from the waste products, the weight of the insect is reduced, which is a important factor for the flight. The bugs are feeding on wild plants, renewing their store of food with the sap and oil. However, the main food reserves in the first division of the midgut, as well as the whole midgut, the reserves of the fat body and the state of the sex glands show no conspicuous alterations.

Towards the second half of October begins the period of hibernation. However, because of the mild climate prevailing at the Kashka-Darya region, the state of diapause does not set in at once. Until the beginning of the rainy winter period the midgut continues functioning, though slowly, and its volume is being reduced not only because of loss of water, but also because the quantity of the food in the 1st division decreases. The 2nd and 3rd divisions grow thin; the rectum is being filled gradually with an increasing quantity of dense black fecalia. There is usually no solid excretion from the main trunks of the Malpighian tubes. The quantity of haemolymph diminishes, the fat body is mostly copious. The odoriferous glands become flat, plicated. The testes continue slowly to develop; they become piriform and intense orange, or red. It is only the ovaries of the female that undergo diapause; they remain simi-

lar to those in the bugs during summer, when they are feeding on crops On account of the development of testes the males seem to consume more of their food reserves in the midgut and of the fat body. Within this period the bug lives largely at the expense of the contents of the midgut and of the part of its fat body. It may, however, also feed actively on the plants available at the place of hibernation. Because of the warm climate prevalent at the valley, the processes described go on more intensely in the bugs dwelling there. Those which hibernate at the mountain soon pass to a condition which may be compared to preservation at a refrigerator.

Along with the rainy period the function of the rectum changes sharply: probably because of the fecalia accumulated in the rectum, the latter collects much water and swells thus filling up the largest part of the abdomen. Now its function is to control the water-balance of the insect, which is a highly important adaptation. The hibernation of the bugs proceeds at this time under conditions of extreme humidity, especially in the valley. They seem, however, to resist it fairly well. They are more sensitive to water deficiency in the organism. During hibernation the nutriment stores in the 1st division of the midgut continue to diminish, the intestine becomes anaemic, pale. The quantity of the fat body also decreases. But a bug that went through a successfull hibernation has still some food in the 1st division of its midgut, and a fat body which is now dirty-grey in colour.

Towards the end of February or the beginning of March in the valley the bugs pass to active life without leaving their winter shelters; at the mountain this takes place within the end of March—middle May. During the first fortnight of March the bugs start moving among their wintering litter; judging by the state of their intestine, they appear to be sucking on young shoots of plants that are sprouting under shelter. The odoriferous glands grow fuller, the testes which are a deep-red by this time, and the accessory glands which have swollen, ripen rapidly. The ovaries which until this time remained underdeveloped, white, short tubules, start growing; there appear shallow constrictions, signs of the developing chambers. The testes increase in volume. The bug evacuates its swollen rectum and is now ready to the flight to plants. The period of entirely passive life is thus of very short duration, being dependent on the length of the cold winter season and on the altitude of the place of hibernation.

As soon as the bugs leave their winter shelter, their odoriferous glands, by whose secretion they are guided when in search for individuals of the same species on wild plants or crops, become intensely active. They start feeding, and thrive rather on wild plants than on young shoots of barley and wheat, their ovary developing more rapidly in the former case.

Since middle April and further on, warm weather being steadily established, the bugs feed intensely, overstuffing the 1st and 3rd division of the midgut which works very energetically. If until this period the nutrition of their sex-glands proceeded at the expense of the fat body and partly at the expense of the stores of nutriments in the midgut, now the fat body takes scarcely any part in the process. The ovaries show a vigorous development, the abdomen is almost filled with green eggs. Bugs that received abundant food during a warmer season, are likely to lay eggs containing a larger amount of yolk and to produce a more resistant progeny. Owing to this, as well as to the higher temperatures that are prevalent at the end of April and during May, the eggs and larvae of the bugs that attack the valley crops later, display a higher rate of development.

When planning to control the pest on cultivated crops it is necessary to know the numerical fluctuations it is subject to. As the bugs newly arrived from the mountains have underdeveloped ovaries in the shape of short white tubules, it is no difficult matter to get aware of their arrival from the mountains without having recourse to examination of their wintering places in the highlands.

A bug which has completed its propagation activity and approaches natural death, shows the following characters. Its body cavity is very poor in water, the bug becomes dry, its intestine is shrivelled and earthy in colour. Which

has been left from the fat body is flabby, semi-transparent, dirty-grey; the odoriferous glands are pale, flat, almost transparent. The ovary is exhausted, flabby, white, with irregular constrictions; there occur individual green egg-cells which have not completed their development. The testes are diminutive, flat, dull.

What is remarkable for the internal organs of *Eurygaster integriceps*, is the extreme stability of its organization. Out of the 721 bugs dissected and examinede, in only one there were found six egg-tubes at either side instead of the usual number seven; only one has been found to have two main trunks of Malpighian tubes instead of the usual four; and only in one the 2nd and 3rd divisions of the midgut have been found to be situated in the left part, instead of the right one.

To sum up, as internal reserve for the winter, *Eurygaster integriceps* makes use of its fat body and of the food stored in the 1st division of the midgut before the insect has left the crops. Under favourable conditions these stores suffice until the spring awaking. The climate prevalent in the Kashka-Darya valley permits of additional feeding, until the cold season sets in, in autumn, and under the winter shelters, early in spring. The intestine is functioning at a low rate almost throughout the period after the bugs leave the crops until they awake in spring, except for a rather short period of cold, when they fall into complete torpidity. Only the female reproductive system undergoes a complete diapause; the development of the testes, which proceeds in the male during hibernation, calls forth a considerable consumption of nutrient material, as a result of which they are more exhausted than the females towards the close of hibernation An important adaptation which makes the bugs more resistant against the excess of moisture n the wintering places, consists in that their rectum ceases to eliminate fecalia at the approach of autumn and becomes an apparatus controlling the insect's water balance. In spring the bugs feed better on wild plants than on cultivated crops. The odoriferous glands play an important rôle, since it seems almost beyond any doubt that the bugs gather together being allured by their scent. The natural conditions prevailing in the Kashka-Darya region are favourable for *Eurygaster integriceps*; the high degree of adaptation of the pest makes it the more injurious in this region.

Laboratory for Morphology of Invertebrates.
A. N. Sewertzoff Institute of Evolutionary Morphology.
Academy of Sciences of the USSR.

Received
14.II.1943.

Prix $ 0.50